

國立中央大學 113 學年度碩士班考試入學試題

所別： 統計研究所碩士班 不分組(一般生)

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統計研究所碩士班 不分組(在職生)

科目： 基礎數學

\* 本科考試可使用計算器，廠牌、功能不拘

計算題應詳列計算過程，無計算過程者不予計分

1. Calculate

$$(a) \int_{-\infty}^{\infty} x^2 e^{-(x-1)^2/2} dx, \quad (b) \int_0^1 x^5 (1-x)^6 dx. \quad (9+9=18\%)$$

2. Test for convergence (write down the reason):

$$(a) \sum_{n=1}^{\infty} \frac{1}{n^2+1}, \quad (b) \sum_{n=1}^{\infty} \frac{3^n}{n \cdot 5^n}. \quad (8+8=16\%)$$

3. Find the minimum and maximum value of  $x^2 + y^2 + z^2$  subject to

$$\text{the constraint conditions } \frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1 \text{ and } z = x + y. \quad (16\%)$$

4. Let (16%)

$$A = \begin{bmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & 1 \end{bmatrix}.$$

Find  $A^{10}$ . (Hint: Diagonalize  $A$  s.t.  $A = PAP^{-1}$ )

5. (a) Suppose that (9+9=18%)

$$A = \begin{pmatrix} A_{11} & C \\ 0 & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & 0 \\ C & B_{22} \end{pmatrix}$$

where  $A_{11}, A_{22}, B_{11}$  and  $B_{22}$  are invertible matrices.

Verify that

$$A^{-1} = \begin{pmatrix} A_{11}^{-1} & -A_{11}^{-1}CA_{22}^{-1} \\ 0 & A_{22}^{-1} \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} B_{11}^{-1} & 0 \\ -B_{11}^{-1}CB_{22}^{-1} & B_{22}^{-1} \end{pmatrix}$$

(b) Derive the inverse of the following matrix

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 3 & 7 & 2 & 3 \\ 2 & 5 & 1 & 2 \end{pmatrix}$$

6. Suppose that  $\lambda$  is the eigenvalue of the square matrix  $A$ . (8+8=16%)

(a) Verify that  $\lambda^k$  is the eigenvalue of  $A^k$ ,  $k$  is a positive integer.

(b) If  $X$  is the eigenvector of  $A$  associated with  $\lambda$ , then what is the eigenvector of  $A^k$  associated with  $\lambda^k$ .