

考試科目	微積分	系所別	企業管理研究所(乙組)	考試時間	2月6日(二) 第四節
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Show all your work to earn the credits.

1. Evaluate the following limits:

(a) (10 points) $\lim_{x \rightarrow 1} \left[(x-1)^2 \sin\left(\frac{2}{x-1}\right) \right]$

(b) (10 points) $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{n-1}{n}\right)^2 \right]$

2. Evaluate the following:

(a) (10 points) $\frac{d}{dx} \int_{-\ln x}^{\ln x} \sin(e^t) dt$

(b) (10 points) $\frac{d}{dx} [\cos(2x)]^{\sin(3x)}$

3. Evaluate each of the following integrals:

(a) (10 points) $\int \frac{1}{(x^2+1)^2} dx$

(b) (10 points) $\int \frac{3}{1+e^{-3x}} dx$

4. (10 points) Determine whether the following series

$$\sum_{n=2}^{\infty} (-1)^n \ln\left(\frac{n+1}{n}\right)$$

is conditionally convergent, absolutely convergent or divergent. Show your work and state which test you use.

5. Given the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (x-1)^n}{n}$$

(a) (4 points) Find all the values of x at which the power series converges.

(b) (3 points) Find the eighth-order derivative $f^{(8)}(1)$ at $x = 1$.

(c) (3 points) Evaluate the sum of the alternating harmonic series:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$$

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- 一、作答於試題上者, 不予計分。
二、試題請隨卷繳交。

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6. A retailer has been selling 1200 tablet computers a week at \$350 each. The marketing department estimates that an additional 80 tablets will sell each week for every \$10 that the price is lowered. Let x be the number of tablets sold.

- (a) (2 points) Find the demand function.
- (b) (3 points) What should the price be set at in order to maximize revenue?
- (c) (3 points) If the retailer's weekly cost function is

$$C(x) = 35,000 + 120x$$

What is the marginal profit function?

- (d) (2 points) What additional profit is realized if the sale of tablets is increased from 1200 to 1201 per week?

7. The quantity x demanded each week is related to the unit price p by the demand equation

$$x = \sqrt{169 - p^2}, \quad 0 \leq p \leq 13.$$

- (a) (4 points) Find the price elasticity of demand that corresponds to $p = 5$.
- (b) (2 points) Is the demand elastic or inelastic when $p = 5$?
- (c) (2 points) For what price p is the demand unitary?
- (d) (2 points) If the unit price is increased slightly from 5, will the revenue increase or decrease?

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