# 國立臺北大學 113 學年度碩士班一般入學考試試題

系（所）組別：統計學系

科　　　目：基礎數學



☐可 ☑不可使用計算機

**第一大題**（共 50%，計算題請敘述計算過程，無計算過程不給分）

1. Let $A = [a_{ij}]$ be an $m \times n$ matrices, $B = [b_{ij}]$ be an $m \times m$ matrices and $\lambda$ be a given constant. Use Sigma notation ( i.e. $\Sigma$ ) and $a_{ij}, b_{ij}$ to represent $trace(\lambda I - A^T BA)$. (10%)

2. Let $A = \begin{bmatrix} 1 & -1 & 2 & 1 & 1 \\ 1 & -1 & 2 & 2 & 1 \\ 1 & -1 & 4 & 1 & 1 \\ 1 & -2 & 2 & 1 & 1 \\ -2 & 2 & -4 & -2 & 1 \end{bmatrix}$. Find the inverse of $A$. (10%)

3. Define $T: P_2 \to P_3$ by $T\big(p(x)\big) = xp(x)$.
   (a) Prove that $T$ is linear. (7%)
   (b) Let $B = \{1 + 2x, 1 - x\}$ and $C = \{1, x, x^2 + 1\}$. Determine $[T]_{C \leftarrow B}$ , i.e. the matrix of $T$ with respect to bases $B$ and $C$. (7%)
   (c) Determine the rank and nullity of $T$. (6%)

4. Let $\mathrm{S} = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_\mathrm{n}\}$ be a basis for $\mathrm{R^n}$ and A be a $\mathrm{n \times n}$ nonsingular matrix. Please show that $\{\mathrm{A}\vec{v}_1, \mathrm{A}\vec{v}_2, \dots, \mathrm{A}\vec{v}_\mathrm{n}\}$ is linear independent. (10%)

**第二大題**（共 50%）

1. Calculate the $\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx$. (15%)

2. Let the $f(\theta) = \frac{n!}{(n-x)!x!}\theta^x(1-\theta)^{n-x}$, $x < n$, be the function of $\theta \in [0, 1]$.

   Denoted $F(\theta) = \log f(\theta)$, where x and n are the positive, fixed constants. Show that $x/n$ is the unique critical point that is a maximizer of $F$ concerning $\theta$. (15%)

3. Find the $\lim\limits_{x \to +\infty} \frac{e - (1+x)^{\frac{1}{x}}}{x}$. (10%)

4. Calculate the $\int_0^1 (4x^3 - 3x^2) dx$ by definition of Riemann sum.

   Hint: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$. (10%)

試題隨卷繳交