國立臺灣科技大學
113學年度碩士班招生

試題

系所組別：1500資訊工程系碩士班

科　　目：計算機數學

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（總分為 100 分；所有試題務必於答案卷內頁依序作答）

1. [15%] Answer the following questions concerning graph and its associated time complexity issues. Use big-O notation, with n representing the number of nodes, m representing the number of edges, and $deg(u)$ indicating the number of neighbors of node u in the graph.
   (1) (5%) What are the upper bound and lower bound of m in a connected graph?
   (2) (2%) In the following questions, when comparing graph implementations using *Adjacency Matrix* and *Adjacency List*, please enumerate their time complexities for *finding an edge* and specify which implementation is faster.
   (3) (2%) Enumerate their time complexities for *finding the degree of node* and specify which implementation is faster.
   (4) (2%) Enumerate their time complexities for *traversing the graph* and specify which implementation is faster.
   (5) (2%) Enumerate their space complexities for *representing of a sparse graph* and specify which implementation requires less storage.
   (6) (2%) Enumerate their space complexities for *representing of a dense graph* and specify which implementation requires less storage.

2. [10%] Solving the problem of scheduling n jobs on a single processor. Each job j requires processing time t_j without interruption, and has a deadline d_j. If job j starts at time s_j, it will be finished at $f_j = s_j + t_j$. The *lateness* of the job j measures how long it finishes after its deadline, max $\{0, f_j - d_j\}$. The goal is scheduling all jobs to minimize the maximum lateness of a job among the n jobs. Suppose we have six jobs with specified required times and deadlines, as shown in the table below. Please provide the job scheduling using *greedy algorithm*.

	Job 1	Job 2	Job 3	Job 4	Job 5	Job 6
t_j	2	4	2	1	3	3
d_j	7	9	14	8	5	13

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3. [25%] There are 8 boys and 6 girls in a tennis club. The IDs of boys are #1 ~ #8, and the IDs of girls are #9 ~ #14. Please answer the following questions.
   (1) (5%) If these 14 members sit around a round table, what is the probability of "#1 and #2 are not adjacent"?
   (2) (6%) How many different ways can these 6 girls be divided into three groups?
   (3) (6%) How many possible ways can we select 3 members from this club without any consecutive IDs?
   (4) (8%) These 14 members are divided into two teams, where Team A has 4 boys and 2 girls, and the remaining 4 boys and 4 girls are in Team B. Suppose each team selects two members for a contest. A random variable X is defined as the total number of boys selected. What is the distribution of this random variable?

4. [10%] Let $A = \begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & 0 \\ 0 & 3\sqrt{5} \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$.
   (1) (5%) Write down the matrix that is the best rank-1 approximation of A.
   (2) (5%) Are A's eigenvalues $\sqrt{5}$ and $3\sqrt{5}$?

5. [15%] True or False.
   (1) (3%) Let A be an $m \times n$ matrix, then the matrices A^TA and AA^T have the same set of nonnegative eigenvalues.
   (2) (3%) Let P be the transition matrix of a Markov chain, then P is orthogonal.
   (3) (3%) Let A be a diagonalizable square matrix, then there is a unique invertible matrix P and a diagonal matrix D such that $AP = PD$.
   (4) (3%) Let L be a linear transformation and $\{x_1, x_2, \dots, x_k\}$ be a linearly independent vector set of the domain of L, then the set $\{L(x_1), L(x_2), \dots, L(x_k)\}$ is also linearly independent.
   (5) (3%) Let $Ax = b$ be an overdetermined linear system, then it is possible that this linear system has multiple least squares solutions.

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6. [5%] A linear transformation T is defined by $T(x) = x + C$, where C is a constant vector. What conclusion can you draw from T?

7. [10%] Answer the following questions.
   (1) (5%) Write down the conditions for an $n \times n$ square matrix A to have zero or one as one of its eigenvalues.
   (2) (5%) Given the aforementioned conditions in (1), compute $\det(A^2 - A)$, where det denotes the determinant of matrices.

8. [10%] A linear operator is defined by $T(x) = Ax$, where A is a 2×2 matrix and $x \in \mathbb{R}^2$. If T maps every rectangle to some segment, what is the rank of T? Why?