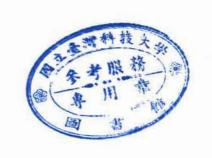# 國立臺灣科技大學
113學年度碩士班招生
試題

系所組別：0420材料科學與工程系碩士班乙組

科　　　目：工程數學

<<504202>>

國立臺灣科技大學 113 學年度碩士班招生試題

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(總分為 100 分；所有試題務必於答案卷內頁依序作答)

1. (14%) The given first order differential equation (ODE) is
$$\frac{dy}{dx} = \frac{y - 4x}{x - y}.$$
   (1) (2%) Is the ODE linear? (Show your reason)
   (2) (2%) Is the ODE exact? (Show your calculation and reason)
   (3) (2%) Does the ODE have the integrating factor $I(x)$ or $I(y)$? (Show your calculation)
   (4) (8%) Find the general solution of the ODE

2. (6%) The given linear second order ODE is
$$y'' + 4y = 0.$$
   and the general solution is $y = c_1 \cos(2x) + c_2 \sin(2x)$. With the boundary condition $y(0) = -2$ and $y(2\pi) = -2$, is the solution of the ODE one solution, no solutions, or infinitely many solutions? (Show your calculation)

3. (10%) The given linear second order ODE is
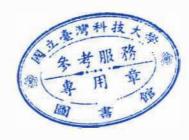$$(x - 1)y'' - xy' + y = 0.$$
   The ODE has one homogeneous solution e^x. Find out the other homogeneous solution by the reduction of order.

4. (20%) Use Laplace Transform to solve the given ODE
$$y' + y = f(t), \quad y(0) = 1$$
$$f(t) = \begin{cases} t & 0 \le t < 2 \\ 1 & t \ge 2 \end{cases}$$
   (1) (3%) Using unit step function to express $f(t)$
   (2) (5%) Find the Laplace transform of $f(t)$
   (3) (12%) Find the solution of the ODE

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(總分為 100 分；所有試題務必於答案卷內頁依序作答)

5. (10%) Use the Gaussian elimination to solve the given system.

$x_1 + x_3 - x_4 = 1$

$2x_2 + x_3 + x_4 = 3$

$x_1 - x_2 + x_4 = -1$

$x_1 + x_2 + x_3 + x_4 = 2$

6. (10%) Solve the given system of equations by Cramer's rule.

$x_1 + 2x_2 + x_3 = 8$

$2x_1 - 2x_2 + 2x_3 = 7$

$x_1 - 4x_2 + 3x_3 = 1$

7. (15%) Find the eigenvalues and eigenvectors of the given matrix.

$$\begin{bmatrix} 1 & 6 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

8. (15%) Find the length of the curve traced by the given vector function on the indicated interval.

$\mathbf{r}(t) = a \cos t\, \mathbf{i} + a \sin t\, \mathbf{j} + ct\, \mathbf{k}\,;\, 0 \leq t \leq 2\pi$