國立臺灣科技大學
113學年度碩士班招生
試題

系所組別：0330機械工程系碩士班丙組

科　　目：工程數學

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(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. (10%) Solve the following ordinary differential equation:

$$\begin{aligned} \frac{d^2y}{dx^2} - m^2 y &= 0 \\ y(0) &= T_b \\ \frac{dy(L)}{dx} &= 0 \end{aligned}$$

where m, T_b, and L are constants.

2. (20%) Solve the following initial value problem:

$$\begin{aligned} \frac{dy_1}{dx} &= 2y_1 + 2y_2 \\ \frac{dy_2}{dx} &= 5y_1 - y_2 \\ y_1(0) &= 0 \\ y_2(0) &= 7 \end{aligned}$$

3. (10%) Consider the function defined below:

$$y(t) = \begin{cases} 1, & 0 \le t < 1, \\ -1 & -1 \le t < 0. \end{cases}$$

(a) Find the Fourier series of $y(t)$. (5%)

(b) Determine the value of the sum of the following infinite series (5%):

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = ?$$

4. (20%) Consider the **Sturm-Liouville system**:

$$\begin{aligned} y'' + \lambda y &= 0, \qquad \lambda > 0, \qquad 0 < x < 1 \\ y(0) &= 0, \\ y(1) + y'(1) &= 0. \end{aligned}$$

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(a) Find the equation for determining the eigenvalues of the Sturm-Liouville system. You need **NOT** calculate the values of the eigenvalues. (5%)

(b) Please specify the weighting function of the orthogonality relationship. (5%)

(c) Show that the eigenfunctions corresponding to two distinct eigenvalues are orthogonal to each other. (10%)

5. (20%) Solve the following second order partial differential equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2}, \qquad t > 0 \text{ and } 0 < y < 1$$

subject to the initial condition:

$$u(0, y) = 0, \qquad 0 \leq y \leq 1$$

and the following boundary conditions:

$$u(t, 0) = 1, \qquad u(t, 1) = 0, \qquad t > 0.$$

(a) Determine if the type of the PDE is **wave equation, diffusion equation**, or **equilibrium equation. (5%)**

(b) Find the **steady state solution. (5%)**

(c) Find the **transient solution. (10%)**

6. (20%) Solve the following transient heat equation for the domain where $x > 0$ using the Laplace transform:

$$\begin{aligned} \frac{\partial^2 T(x,t)}{\partial x^2} &= \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} \\ -k\frac{\partial T(x=0,t)}{\partial x} &= h\left[T_\infty - T(x=0,t)\right] \\ T(x \to \infty, t) &= 0 \\ T(x, t=0) &= 0 \end{aligned}$$

where α is heat diffusivity (constant), k is thermal conductivity (constant), T_∞ is ambient temperature (constant), and h is convection coefficient (constant).

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$\overline{F}(s)$	$F(t)$
$\frac{1}{s^2}e^{-ks}$	$(t-k)\,U\,(t-k)$
1	$\delta(t)$ $[\delta(t) \equiv \text{delta function}]$
e^{-as}	$\delta(t-a)$
$\frac{1}{s}e^{-k/s}$	$J_0\left(2\sqrt{kt}\right)$
$\frac{1}{s^\mu}e^{-k/s}\quad(\mu>0)$	$\left(\frac{t}{k}\right)^{(\mu-1)/2} J_{\mu-1}\left(2\sqrt{kt}\right)$
$\frac{1}{s^\mu}e^{k/s}\quad(\mu>0)$	$\left(\frac{t}{k}\right)^{(\mu-1)/2} I_{\mu-1}\left(2\sqrt{kt}\right)$
$e^{-k\sqrt{s}}\quad(k>0)$	$\frac{k}{2\sqrt{\pi t^3}}\exp\left(-\frac{k^2}{4t}\right)$
$\frac{1-e^{-k\sqrt{s}}}{s}\quad(k\geq 0)$	$\text{erf}\left(\frac{k}{2\sqrt{t}}\right)$
$\frac{1}{s}e^{-k\sqrt{s}}\quad(k\geq 0)$	$\text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
$\frac{1}{\sqrt{s}}e^{-k\sqrt{s}}\quad(k\geq 0)$	$\frac{1}{\sqrt{\pi t}}\exp\left(-\frac{k^2}{4t}\right)$
$\frac{1}{s^{3/2}}e^{-k\sqrt{s}}\quad(k\geq 0)$	$2\sqrt{\frac{t}{\pi}}\exp\left(-\frac{k^2}{4t}\right)-k\text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
$\frac{1}{s^{1+n/2}}e^{-k\sqrt{s}}(n=0,1,2,\ldots,k\geq 0)$	$(4t)^{n/2}i^n\text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
$\frac{e^{-k\sqrt{s}}}{a+\sqrt{s}}\quad(k\geq 0)$	$\frac{1}{\sqrt{\pi t}}\exp\left(-\frac{k^2}{4t}\right)$ $-ae^{ak}e^{a^2t}\text{erfc}\left(a\sqrt{t}+\frac{k}{2\sqrt{t}}\right)$
$\frac{e^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})}\quad(k\geq 0)$	$e^{ak}e^{a^2t}\text{erfc}\left(a\sqrt{t}+\frac{k}{2\sqrt{t}}\right)$
$\frac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}}\quad(k\geq 0)$	$e^{-at/2}I_0\left(\frac{1}{2}a\sqrt{t^2-k^2}\right)U(t-k)$
$\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}\quad(k\geq 0)$	$J_0\left(a\sqrt{t^2-k^2}\right)U(t-k)$
$\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2-a^2}}\quad(k\geq 0)$	$I_0\left(a\sqrt{t^2-k^2}\right)U(t-k)$
$\frac{ae^{-k\sqrt{s}}}{s(a+\sqrt{s})}\quad(k\geq 0)$	$-e^{ak}e^{a^2t}\text{erfc}\left(a\sqrt{t}+\frac{k}{2\sqrt{t}}\right)$ $+\text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$
$\frac{1}{s^2}e^{-k\sqrt{s}}$	$\left(t+\frac{k^2}{2}\right)\text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$ $-k\left(\frac{t}{\pi}\right)^{1/2}\exp\left(-\frac{k^2}{4t}\right)$