

國立中正大學

113 學年度碩士班招生考試

試題

[第 1 節]

科目名稱	工程數學
系所組別	機械工程學系-乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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科目名稱：工程數學

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系所組別：機械工程學系-乙組

1. (5%) Please verify \vec{v}_1 , \vec{v}_2 , \vec{v}_3 are linear independent, where $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$.

2. (25%) There is a simultaneous homogeneous differential equations $X' = AX$, where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$,

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

(1) (20%) Please find the eigenvalues and eigenvectors of matrix A .

(2) (5%) Please find the general solution of simultaneous homogeneous differential equations $X' = AX$.

3. (20%) There is a periodic function $f(x) = x^2$, $(-\pi < x < \pi)$ and $f(x) = f(x + 2\pi)$.

(1) (10%) Please find the Fourier series of $f(x)$.

(2) (10%) Please use the Parseval equality to prove the following equality.

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots = \frac{1}{90} \pi^4$$

4. (25%) Consider the following differential equation

$$\frac{dx}{dt} + 3x = e^{-2t}, \quad x(0) = 1$$

(1) (10%) Please find the homogeneous solution.

(2) (15%) Please find the exact solution for $x(t)$ without using Laplace transform.

5. (25%) Consider the following differential equation

$$2 \frac{dx_1}{dt} - \frac{dx_2}{dt} + 3x_1 = 1; \quad \frac{dx_1}{dt} + \frac{dx_2}{dt} - 4x_2 = e^{-t}, \quad x_1(0) = 1; x_2(0) = 0$$

(1) (5%) Let the Laplace transform of $x_1(t)$ be $X_1(s)$. Please express the Laplace transform of $\frac{dx_1}{dt}$ in terms of $X_1(s)$.

(2) (20%) Please find the Laplace transform of $x_1(t)$ and $x_2(t)$ directly from the differential equation.