

國立中正大學

113 學年度碩士班招生考試

試題

[第 2 節]

科目名稱	線性代數與微分方程
系所組別	電磁晶片組 電機工程學系- 計算機工程組 電力與電能處理甲組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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科目名稱：線性代數與微分方程

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系所組別：電機工程學系-電磁晶片組、計算機工程組、電力與電能處理甲組

第一部分：線性代數

1. Let $C = \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $E = \begin{bmatrix} 0 & a \\ 1 & b \end{bmatrix}$. Answer the following questions with the appropriate matrix names (C , D or E). Note: No partial scores are given for each question.
 - a. (5 pts.) Identify matrices that are row equivalent when a is 1 and b is 0.
 - b. (5 pts.) Determine which matrix has $\{0\}$ as the orthogonal complement of its row space when a is 0 and b is 1.
 - c. (5 pts.) Which of these matrices satisfies the condition that the rank plus the nullity equals 2?
 - d. (5 pts.) Which matrix is not full-rank when a and b are 1's?

2. (10 pts.) For a given matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, identify all possible matrices B , where the last row of B is 0 , such that the determinants of AB and BA are equal.

3. Let $T_1 : P_1 \rightarrow P_2$ be the linear transformation defined by $T_1(p(x)) = x \cdot p(x)$ and let $T_2 : P_n \rightarrow P_n$ be the linear operator defined by $T_2(p(x)) = p(x + 1)$, where $B = \{1, 2x\}$ and $B' = \{1, x, 2x^2\}$ are bases for P_1 and P_2 , respectively. Every case requires detailed information.
 - a. (10 pts.) Represent the linear transformation T_1 from P_1 to P_2 as the matrix $[T_1]_{B \rightarrow B'}$.
 - b. (10 pts.) Show the matrix representation of the composition of two linear transformations $[T_2 \circ T_1]_{B \rightarrow B'}$.

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第二部分：微分方程

4. Select the correct answer (Single) for the following problems.

(1) The differential equation $y'' + 2yy' + 3y = 0$ is (5 pts.)

- a. first order linear
- b. second order linear
- c. third order linear
- d. first order nonlinear
- e. second order nonlinear

(2) The differential equation $(x^2 + y^2)y' = xy$ is (5 pts.)

- a. linear
- b. homogeneous
- c. separable
- d. exact
- e. Bernoulli

5. (10 pts.) Solve the DE problem by undetermined coefficients.

$$y'' + y = 8 \cos 2x - 4 \sin x, \quad y\left(\frac{\pi}{2}\right) = -1, \quad y'\left(\frac{\pi}{2}\right) = 0$$

6. (10 pts.) Use Laplace transform to solve the given initial-value problem.

$$y' + y = e^{-3t} \cos 2t, \quad y(0) = 0$$

7. (10 pts.) Solve the following DE by systematic elimination.

$$\frac{dx}{dt} = -5x - y$$

$$\frac{dy}{dt} = 4x - y$$

$$x(1) = 0, \quad y(1) = 1$$

8. (10 pts.) Solve the following DE problem by variation of parameters.

$$y''' + 4y' = \sec 2x$$