國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】

－作答注意事項－

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學113學年度碩士班暨碩士在職專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

一、選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) A box contains x white balls and 30 black balls. If a ball is randomly drawn from the box, the probability of drawing a white ball is $\frac{1}{x}$. Find the value of x.
   (A) 3
   (B) 5
   (C) 6
   (D) 10
   (E) 30

2. (5%) There are s boys and t girls in a class. If a team of 2 boys and 3 girls are selected from the class to participate in a competition, how many different teams can be formed?
   (A) $6st$
   (B) $\frac{st}{6}$
   (C) $6st(s-1)(t-1)(t-2)$
   (D) $\frac{st(s-1)(t-1)(t-2)}{6}$
   (E) $\frac{st(s-1)(t-1)(t-2)}{12}$

3. (5%) Suppose that A and B are two independent events. Which of the following is correct?
   (A) A and B must be mutually exclusive.
   (B) $P(A)+P(B)=1$
   (C) $P(A\cup B)=P(A)P(B)$
   (D) $P(A\cap B)=P(A)P(B)$
   (E) None of the above.

4. (5%) Suppose that the probability distribution of a continuous random variable X is memoryless. Let $p,q\geq 0$. Which of the following is correct?
   (A) $P(X<p+q|X<p)=P(X<q)$
   (B) $P(X>p+q|X>p)=P(X>q)$
   (C) $P(X<p+q|X<p)=P(X<p)$
   (D) $P(X>p+q|X>p)=P(X>p)$
   (E) None of the above.

5. (5%) The number of customer arrivals in a working hour of a bank follows the Poisson distribution with mean 8.8. What is the probability that there are not more than 3 arrivals in a working hour?
   (A) 0.0171
   (B) 0.0230
   (C) 0.0244
   (D) 0.0416
   (E) None of the above.

6. (5%) Let T be the packet transmission time at a router. If T follows the exponential distribution with non-zero mean $\frac{1}{\lambda}$, what is the median of T?

(A) $\frac{\ln 2}{\lambda}$

(B) $\frac{\lambda}{\ln 2}$

(C) $\frac{2}{\lambda}$

(D) $\frac{1}{\lambda}$

(E) λ

7. (5%) Define $E(X) = \frac{3}{4}$, $E(Y) = \frac{1}{2}$, and $E(XY) = \frac{3}{14}$. Find σ_{XY}.

(A) $-\frac{33}{56}$

(B) $-\frac{9}{56}$

(C) 0

(D) $\frac{9}{56}$

(E) $\frac{33}{56}$

8. (5%) Let X and Y be two random variables such that $\sigma_{XY} = 0$. Which of the following is correct?

(A) X and Y are uncorrelated.

(B) X and Y are independent.

(C) $X = 0$ or $Y = 0$

(D) $E(X) = 0$ or $E(Y) = 0$

(E) $\sigma_X = 0$ or $\sigma_Y = 0$

9. (5%) Let A and B be two exponentially distributed random variables with parameters λ_A and λ_B, respectively. That is, $A \sim \text{Exp}(\lambda_A)$ and $B \sim \text{Exp}(\lambda_B)$. Which of the following is true about $\min(A, B)$, the minimum of A and B.

(A) $\min(A, B) \sim \text{Exp}(\min(\lambda_A, \lambda_B))$

(B) $\min(A, B) \sim \text{Exp}(\lambda_A + \lambda_B)$

(C) $\min(A, B) \sim \text{Exp}(\lambda_A \lambda_B)$

(D) $\min(A, B) \sim \text{Exp}(|\lambda_A - \lambda_B|)$

(E) None of the above.

10. (5%) Denote x and y as two real numbers. Let U and V be two independent random variables such that their probability density functions are $f_U(x)$ and $f_V(x)$, respectively. Which of the following is correct?

(A) $f_{UV}(x,y) = f_U(x) * f_V(y)$

(B) $f_{UV}(x,y) = f_U(x+y) * f_V(x+y)$

(C) $f_{UV}(x,y) = \frac{f_U(x)}{f_V(y)}$

(D) $f_{UV}(x,y) = f_U(x) f_V(y)$

(E) None of the above.

二、問答計算題:

1. (10%) Consider a four-point sample space $\Omega = \{w_1, w_2, w_3, w_4\}$ with probabilities assigned to the sample events as given by

$$P(\{w_1\}) = \frac{1}{2},\ P(\{w_2\}) = \frac{1}{4},\ P(\{w_3\}) = \frac{1}{8},\ P(\{w_4\}) = \frac{1}{8}.$$

Define random variables (RVs) X and Y as

$$X(w_1) = 1,\ X(w_2) = 1,\ X(w_3) = 2,\ X(w_4) = 3,$$

$$Y(w_1) = 3,\ Y(w_2) = 3,\ Y(w_3) = 1,\ Y(w_4) = 1.$$

Determine the distribution function of X. Is it the same for Y?

2. (10%) Assume that two RVs X and Y are related by

$$Y = \cos X.$$

Let the PDF of X be given by

$$f_X(x) = \begin{cases} \frac{1}{2\pi}, & -\pi < x < \pi \\ 0, & \text{elsewhere.} \end{cases}$$

Determine the covariance of X and Y.

3. (10%) Consider the two RVs X and Y with the joint PDF:

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}.$$

Let Z be a complex-valued RV, and define $Z = X + jY$. Determine the joint PDF of Z and its complex conjugate Z^*.

4. (20%) Let $g(x)$ be the Gaussian probability density function (PDF) given by

$$g(x)=\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}},$$

and let $h(x)$ be an antisymmetric function shown in Fig. 1. Define a function expressed as

$$f(x_1,x_2)=g(x_1)g(x_2)+h(x_1)h(x_2).$$

Please provide your reasons for answering the following questions.

(a). (5%) Is $f(x_1,x_2)$ a well-defined joint PDF of the RVs X_1 and X_2 ?

(b). (5%) Whether X_1 and X_2 are jointly Gaussian?

(c). (5%) Is the marginal PDF of X_1 Gaussian?

(d). (5%) Are X_1 and X_2 independent?

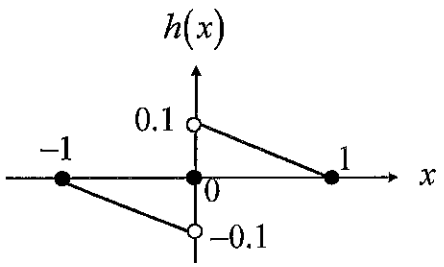

Fig. 1