國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學 113 學年度碩士班暨碩士在職專班招生考試試題

科目名稱：控制系統【電機系碩士班乙組】 題號：431008

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題） 共 3 頁第 1 頁

Problem 1. Consider the state equations (in Jordan Canonical Form) as follows:

$$\dot{x} = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 3 & 0 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 0 \\ 3 & 3 & 0 \end{bmatrix} u, y = \begin{bmatrix} 1 & 0 & b & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} x.$$

(a) Determine the value of a so that the system is not controllable. (5 points)

(b) Determine the value of b so that the system is not observable. (5 points)

Problem 2. The transfer function of a system is given by

$$\frac{Y(s)}{U(s)} = G(s) = \frac{3}{(s-2)(s-4)(s-7)}$$

where U and Y are the Laplace transforms of input u and output y, respectively.

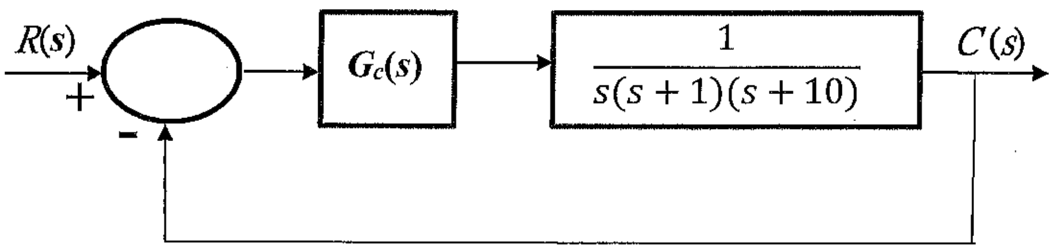
(a) Find the state-space model if $x_1 = y$, $x_2 = \dot{x_1}$ and $x_3 = \dot{x_2}$. (2 points)

(b) Design a state feedback $u = -Kx + v$ with $K = [K_1 \quad K_2 \quad K_3]$ so that the characteristic equation of the closed-loop system is $s^3 + 5s^2 + 8s + 6 = 0$. (4 points)

(c) When use the Luenberger type observer: $\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$ to estimate the real state x, design the observe gain $L = [L_1 \quad L_2 \quad L_3]^T$ so that the roots of characteristic polynomial of the error system are located at $s = -3 \pm j$ and $s = -2$. (4 points)

(d) Use the dynamic output feedback $u = -K\hat{x} + v$ to compensate the system where K is chosen as in problem (b), and $\hat{x}$ is a solution of $\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$ with L as in problem (c). Find the transfer function of the new closed loop system (from v to y). (5 points)

Problem 3. Consider the control system shown below:



(a) Draw the root-locus for the closed-loop with $G_c(s)$ being K. (5 points)

(b) When $G_c(s)$ is a PD compensator, design the compensator for the following time-domain specifications (using approximation and the dominant poles): (10 points)

i) Damping ratio $\zeta = \sqrt{2}/2$.

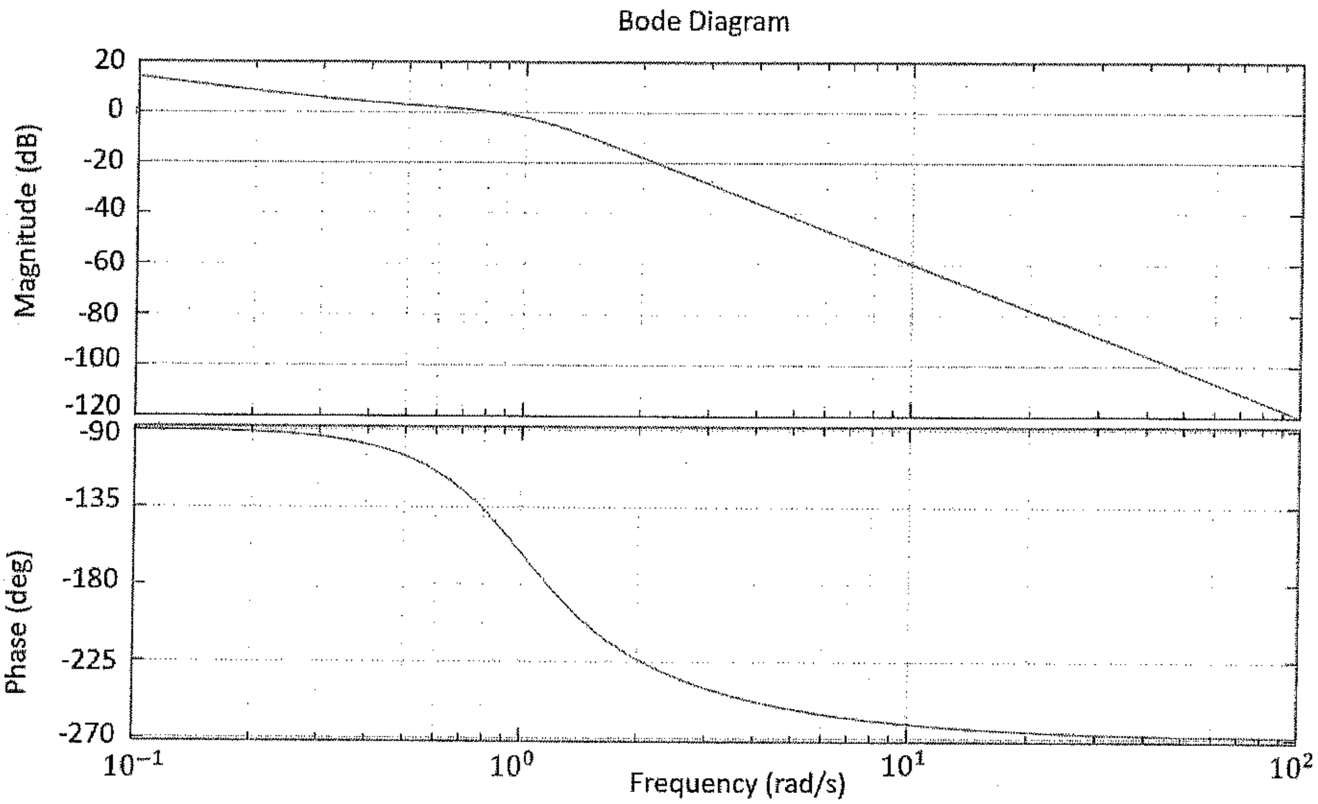
ii) Time constant τ (defined as $1/\zeta\omega_n$) $=1/2$.

Problem 4. A polynomial can be factored as $(s^4 + as^2 + b)(s^6 + cs^4 + ds^2 + e)$ where a, b, c, d and e are constants with $a>0$ and $b>0$. Find the condition on a and b so that $s^4 + as^2 + b$ has just two roots on left half-plane. (5 points) In this case, if we know that $s^6 + cs^4 + ds^2 + e$ has two roots on imaginary axis, find the pair (L, I, R) where L, I, and R are the numbers of its roots located on left half-plane, imaginary axis, and right half-plane, respectively. (5 points)

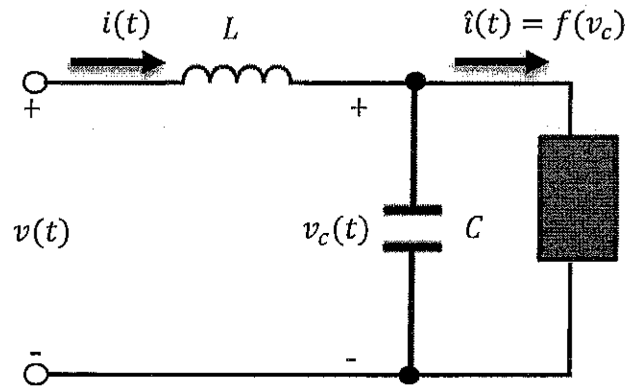
Problem 5. A unit feedback system has the forward-path transfer function $G(s) = \frac{2s^2-s-4K}{s^3+3s^2+2s+6}$. When K=1, find GM. (5 points) Find K>0 such that GM=0. (5 points)

Problem 6. A unit feedback system has the forward-path transfer function $G(s) = \frac{K}{s^2+(a+1)s+a}$. Find the values of K and a to satisfy the following frequency domain specifications: $M_r = 1.05$ and $\omega_r = 12$ rad/sec . (5 points) Calculate the peak percent overshoot of the step response and the bandwidth of the closed-loop system. (5 points)

Problem 7. For a transfer function with the following bode diagram, find PM. (5 points) When the input is $\cos^2(0.5t)$-$\cos(5t)\sin(5t)$, find the steady state output y_{ss}. (5 points)



Problem 8. Consider the following circuit where the block represents a device with the current-voltage characteristic function f. With $x_1 = v_c, x_2 = i$, write down the state space model. (3 points) When $v(t) \equiv 1$ is constant, determine the equilibrium point in case of $f(x)=x^5 - x^3 + 1$. (2 points) Find the linearized system at this equilibrium point. (5 points).



Problem 9. Consider the dynamic equations $\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad y(t) = Cx(t)$ where

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, C = [1 \quad 0 \quad a].$$

Can the system be transferred into controllability canonical form (CCF)? (2 points) Find the all possible values of a so that the system is not observable. (3 points) Find the transformation P so that $x(t) = P\bar{x}(t)$ could transform the state equation into the observability canonical form (OCF) when a=1. (5 points)