國立中山大學 113 學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：高等微積分【應數系碩士班丙組】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

1. (10%) Use $\epsilon - \delta$ argument to show that $\lim_{x\to 1}(x^2+2x)=3$.

2. (10%) Give an example of a function that is NOT integrable on $[0,1]$. Support your argument.

3. (10%) Is the sequence of functions $f_n(x)=\frac{x^n}{1+x^n}$ uniformly convergent on the interval $[0.5,1.5]$? Prove or disprove.

4. (10%) Consider the vector-valued function $G:\mathbb{R}^2\to\mathbb{R}^2$ defined by
$$G(x,y)=(x\cos y, x\sin y).$$
Show that G is locally invertible about every point in $\mathbb{R}^2\setminus\{0\}$.

5. Let $f:\ E\to\mathbb{R}$ be continuous on E, a closed and bounded subset of $\mathbb{R}^n$.
   (a) (5%) Show that the image set $f[E]$ has a maximum point M and a minimum point m.
   (b) (10%) Show that $f[E]$ is a closed and bounded interval.

6. Let $g(x,y,z)=x^3-3xy-y^3-z^2$.
   (a) (6%) Compute the gradient vector $\nabla g(x,y,z)$ and Hessian matrix $H_g(x,y,z)$.
   (b) (4%) Determine if the critical point $(-1,1,0)$ is a local maximum of g.
   (c) (5%) Find the Taylor polynomial of order 2 based at the point $\mathbf{x}_0=(-1,1,0)$.

7. (15%) State and prove one of the two Fundamental Theorems of Calculus.

8. The mean value theorem for integrals states that for a continuous function f defined on $[a,b]$, there exists some $c\in(a,b)$ such that
$$\int_a^b f(t)\,dt=f(c)(b-a).$$
   (a) (6%) Letting $F(x)=\int_a^x f(t)\,dt$, prove the above theorem as an application of the (usual) mean value theorem.
   (b) (9%) Show that the mean value theorem for integrals implies the mean value theorem too, when the function f is continuously differentiable on $[a,b]$.

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