計算題：應詳列計算過程，無計算過程者不予計分。

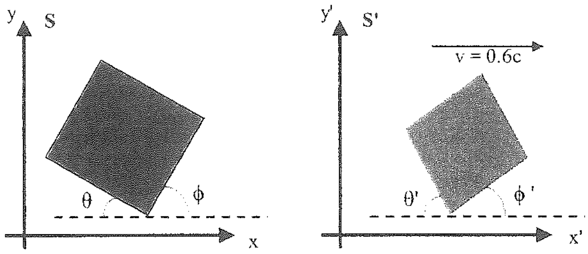
1. A stationary body explodes into two fragments each of mass 3.0 kg that move apart at speeds of 0.8c relative to the original body. Find the mass of the original body. (5%)

2. Two relativistic rockets travel toward each other. As seen from Earth, rocket A, of proper length 500 m, travels with a speed 0.8 c. Rocket B, of proper length 1000 m, has a speed of 0.6 c. The speed of light is defined as c in this equation.

   (a) What is the speed of the rockets relative to each other? (5%)

   (b) An Earth based observer sets his stopwatch to zero as the noses of the rockets pass each other. What will the stopwatch read when the tails of the rockets pass each other? (5%)

3. The shape of a moving square. Consider the square in the xy plane of S with angle θ and ϕ as in following figure. If S' moving with v = 0.6c relative to S in x direction, what are the angle θ' and ϕ' of the figure on S' ? (5%)



4. The surface temperature of the sun is about 5500 °C. Assuming that the sun radiates like an ideal blackbody, at what wavelength does the peak of the solar spectrum occur? The Human body temperate is about 37°C, at what wavelength does the peak of the spectrum occur? (5%)

5. A metal surface illuminated by 8.5×10^{14} Hz light emits electrons whose maximum energy is 0.52 eV. The same surface illuminated by 12×10^{14} Hz light emits electrons whose maximum energy is 1.97 eV. From these data find Plank's constant and the work function of the metal. (10%)

6. The energy levels of a hydrogen atom are quantized. Light with a continuous range of wavelength from 400 nm to 700 nm is shined on the atom. All the light passes by the atom except particular energies that absorbed by the atom, leaving black absorption line in the spectrum. How many absorption lines and what are the wavelength of the absorption line? (10%)

7. For an infinite well with a well width L. The ground energy is E_1. If the well width changes to 2L, and ground energy is E_2. What is the value of E_1/E_2? (5%)

注意：背面有試題

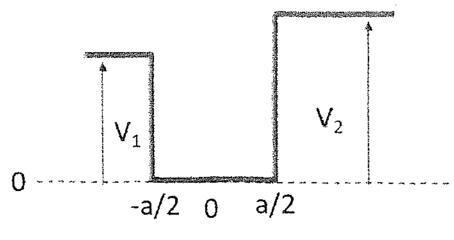
8. In quantum harmonic oscillation system, the ladder operators can be defined as $\hat{a}=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}+\frac{i}{m\omega}\hat{p}_x\right)$; $\hat{a}^+=\sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x}-\frac{i}{m\omega}\hat{p}_x\right)$, where $\hat{p}_x=-i\hbar\frac{d}{dx}$ is the operator of linear momentum. The eigen states of HO system are denoted by $|n\rangle$ with the quantum number n = 0, 1, 2….
   (a) What is the expectation value $\langle p^2\rangle$ for the state $|n=10\rangle$? (5%)
   (b) Proof that the uncertainty $\Delta x\Delta p$ for the HO system is $\Delta x\Delta p=\hbar/2$ for the ground state. (10%)

9. For a rectangular potential barrier, discuss the resonant tunneling effect and the condition of the energy. (5%)

10. Electron bounded to a nucleus is a system which particle with central force. The potential can be given by: $V(r)=Ze^2/4\pi\varepsilon_0 r$. The normalized eigenstates can be solved as $\psi_{n,m,l}(r,\theta,\phi)$ which have energy eigenvalues E_n.
   (a) An electron in hydrogen is excited to total energy E = -1.51 eV. How many different wave function $\psi_{n,m,l}(r,\theta,\phi)$ in which have the same energy? (Ignore spin and fine structure ect.) (5%)
   (b) For $l=3$, what is the minimum value of $(L_x)^2+\left(L_y\right)^2$, where L_x and L_y are x and y components of angular momentum? (5%)

11. Using the Pauli matrices $\sigma_x=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, $\sigma_y=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$, $\sigma_z=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$, find the eigenstate vectors along an arbitrary direction $\boldsymbol{n}=\boldsymbol{a}_x\sin\theta\cos\phi+\boldsymbol{a}_y\sin\theta\sin\phi+\boldsymbol{a}_z\cos\theta$. (10%)

12. Considering a 1D quantum system with potential as shown in the figure, show that the eigenvalue equation is $ka=n\pi+tan^{-1}\left(\frac{\alpha_1}{k}\right)+tan^{-1}\left(\frac{\alpha_2}{k}\right)$, where $k=\sqrt{\frac{2mE}{\hbar^2}}$, $\alpha_1=\sqrt{\frac{2m(V_1-E)}{\hbar^2}}$, $\alpha_2=\sqrt{\frac{2m(V_2-E)}{\hbar^2}}$. (10%)



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