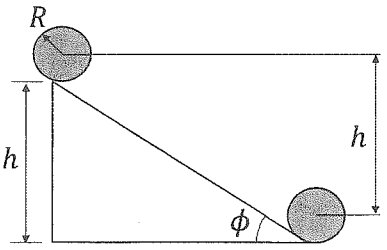
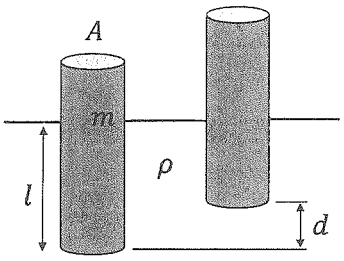
**計算題: 應詳列計算過程，無計算過程者不予計分。**

一 (15%) A round object with mass $m$, radius $R$, and moment of inertia $I$ around its center starts from rest and rolls down an incline with a height $h$ and an angle $\phi$ with the horizontal. (a) (5%) What is the angular velocity of the object when it reaches the bottom of the incline? (b) (5%) How much time does it take to reach the bottom? (c) (5%) What is the instantaneous power the object received from the gravitation field? Suppose the gravitational acceleration is $g$.

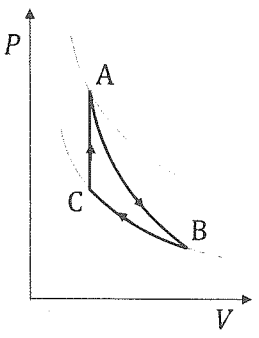



二 (15%) A long rod with cross-sectional area $A$ and mass $m$ floats upright in a fluid of density $\rho$. When it reaches equilibrium under the gravitational field of strength $g$, a length $l$ is immersed in the fluid. (a) (5%) What is the magnitude of $l$? (b) (5%) When the rod is lifted a distance $d$ from equilibrium and then released, it undergoes simple harmonic motion. Determine the angular frequency of the oscillation. (c) (5%) What is the total energy of the rod during the oscillation?



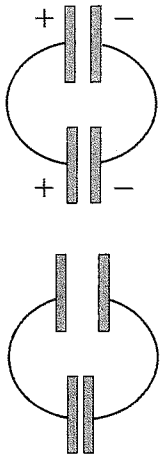
equilibrium

三 (30%) Suppose an $n$ mole of ideal gas has the ratio of molar specific heats $\gamma$. It undergoes three steps to form a heat engine: A → B adiabatic expansion, B → C isothermal compression, C → A isovolumic heating. The volume and temperature at B are $(V_B, T_B)$ and the volume at A is $V_A = V_B/e$, where $e$ is the natural logarithm (自然底數), with $\ln(e) = 1$. The gas constant is $R$. Calculate (a) (5%) the work during B → C, (b) (5%) the change of entropy during B → C, (c) (5%) the temperature at A, (d) (5%) the change of internal energy during C → A, (e) (5%) the work during A → B, and (f) (5%) the efficiency of the engine. Keep notation $e$ in your calculations and do not substitute it by 2.71828 ....

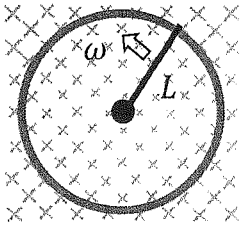



注意：背面有試題

四 (20%) (a) (5%) Given the permittivity of free space $\varepsilon_0$, use Gauss's law to derive the capacitance of a capacitor formed by two parallel plates with a large area $A$ separated by a narrow distance $d$. (b) (5%) Determine the work required to transfer a total charge $Q$ from one plate to the other in such a capacitor when it is initially charge-free. (c) (5%) Connect two capacitors of this type in parallel (upper figure), each with capacitance $C$ and charge $Q$. If the plate separation is increased from $d$ to $3d$ in one capacitor and decreased from $d$ to $d/3$ in the other (bottom figure), express the potential difference on each capacitor in terms of $C$ and $Q$. (d) (5%) In this case, what is the total energy stored in the two capacitors in terms of $C$ and $Q$?




五 (10%) (a) (5%) A conducting bar of length $L$ has one end connected to a pivot, and the other end is connected to a conducting ring around the pivot. A magnetic field is directed perpendicular into the ring and varies with the distance $r$ to the pivot by $B(r) = 2r$. If the rod rotates with a constant angular speed $\omega$ about the pivot, what is the induced motional electromotive force between the ring and the pivot? (b) (5%) If the rod is absent, and the magnetic field changes with both $r$ and time $t$ as $B(r,t) = 2rt$, what is the induced electromotive force along the ring?




六 (10%) An aquarium tank has a depth of 22 cm. (a) (5%) Calculate the apparent depth of the tank when viewed from directly above the water. (b) (5%) If a glass plate, 10 cm thick, is placed on the bottom of the tank, what is the apparent thickness of the plate when viewed from directly above the water? The indices of refraction for air, water, and glass are $n_a = 1$, $n_w = 4/3$, and $n_g = 5/3$, respectively. Express your calculations using fractional numbers (such as 4/3) instead of decimal numbers (such as 1.33…).

注意：背面有試題