類組：物理類　科目：應用數學(2001)　

單選題　不倒扣

1. (5pt) A matrix A is a Hermite matrix, when
   (A) $A^{-1}=A$　(B) $A^{\dagger}=A$　(C) $A^{\dagger}A=I$　(D) $A^{-1}A^{\dagger}=A$　(E) $A^{\dagger}A=A$

2. (5pt) A matrix U is a unitary matrix, when
   (A) $U^{-1}=U$　(B) $U^{\dagger}=U$　(C) $U^{\dagger}U=I$　(D) $U^{-1}U^{\dagger}=U$　(E) $U^{\dagger}U=U$

3. (5pt) An eigen value λ of a Hermite matrix is
   (A) λ is real　(B) λ is pure imaginary　(C) $\lambda\geq 0$　(D) $\lambda\leq 0$　(E) $|\lambda|=1$

4. (5pt) An eigen value λ of a unitary matrix is
   (A) λ is real　(B) λ is pure imaginary　(C) $\lambda\geq 0$　(D) $\lambda\leq 0$　(E) $|\lambda|=1$

5. (5pt) With the use of a unitary matrix U, a Hermite matrix A can be unitary transformed into a diagonal matrix D. Find U and D for Hermite matrix $A=\begin{pmatrix}0&1\\1&0\end{pmatrix}$.

   (A) $U=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&1\end{pmatrix}$, and $D=\begin{pmatrix}1&0\\0&1\end{pmatrix}$　(B) $U=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$, and $D=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

   (C) $U=\frac{1}{\sqrt{2}}\begin{pmatrix}-1&1\\1&1\end{pmatrix}$, and $D=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$　(D) $U=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$, and $D=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$

   (E) $U=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&1\end{pmatrix}$, and $D=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

We have a scaler function $f(x,y,z)$ and a vector function $\vec{F}(x,y,z)$.

6. (5pt) The definition of the gradient, grad$f$ is

   (A) $\text{grad}\, f=\frac{\partial f_x}{\partial x}+\frac{\partial f_y}{\partial y}+\frac{\partial f_z}{\partial z}$　(B) $\text{grad}\, f=\frac{\partial f_x}{\partial x}\vec{e}_x+\frac{\partial f_y}{\partial y}\vec{e}_y+\frac{\partial f_z}{\partial z}\vec{e}_z$

   (C) $\text{grad}\, f=\frac{\partial f}{\partial x}\vec{e}_x+\frac{\partial f}{\partial y}\vec{e}_y+\frac{\partial f}{\partial z}\vec{e}_z$　(D) $\text{grad}\, f=\frac{\partial f}{\partial y}-\frac{\partial f}{\partial z}$

   (E) $\text{grad}\, f=\left(\frac{\partial f_z}{\partial y}-\frac{\partial f_y}{\partial x}\right)\vec{e}_x+\left(\frac{\partial f_x}{\partial z}-\frac{\partial f_z}{\partial x}\right)\vec{e}_y+\left(\frac{\partial f_y}{\partial x}-\frac{\partial f_x}{\partial y}\right)\vec{e}_z$

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7. (5pt) The definition of $\text{curl}\,\vec{F}$ is

(A) $\text{curl}\,\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$　(B) $\text{curl}\,\vec{F} = \frac{\partial F_x}{\partial x}\vec{e}_x + \frac{\partial F_y}{\partial y}\vec{e}_y + \frac{\partial F_z}{\partial z}\vec{e}_z$

(C) $\text{curl}\,\vec{F} = \frac{\partial F}{\partial x}\vec{e}_x + \frac{\partial F}{\partial y}\vec{e}_y + \frac{\partial F}{\partial z}\vec{e}_z$　(D) $\text{curl}\,\vec{F} = \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}$

(E) $\text{curl}\,\vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\vec{e}_z$

8. (5pt) The definition of $\text{div}\,\vec{F}$ is

(A)$\text{div}\,\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$　(B) $\text{div}\,\vec{F} = \frac{\partial F_x}{\partial x}\vec{e}_x + \frac{\partial F_y}{\partial y}\vec{e}_y + \frac{\partial F_z}{\partial z}\vec{e}_z$

(C) $\text{div}\,\vec{F} = \frac{\partial F}{\partial x}\vec{e}_x + \frac{\partial F}{\partial y}\vec{e}_y + \frac{\partial F}{\partial z}\vec{e}_z$　(D) $\text{div}\,\vec{F} = \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}$

(E) $\text{div}\,\vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial x}\right)\vec{e}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\vec{e}_z$

Hereafter $\vec{r}$ is a radial vector defined by $\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$, and $r = |\vec{r}|$.

9. (5pt) Find $\text{grad}\, r$

(A) $\text{grad}\, r = 0$　(B) $\text{grad}\, r = \vec{r}$　(C) $\text{grad}\, r = \vec{r}/r$

(D) $\text{grad}\, r = 3$　(E) $\text{grad}\, r = 3\vec{r}$

10. (5pt) Find $\text{curl}\,\vec{r}$

(A) $\text{curl}\,\vec{r} = 0$　(B) $\text{curl}\,\vec{r} = \vec{r}$　(C) $\text{curl}\,\vec{r} = \vec{r}/r$

(D) $\text{curl}\,\vec{r} = 3$　(E) $\text{curl}\,\vec{r} = 3\vec{r}$

11. (5pt) Find $\text{div}\,\vec{r}$

(A) $\text{div}\,\vec{r} = 0$　(B) $\text{div}\,\vec{r} = \vec{r}$　(C) $\text{div}\,\vec{r} = \vec{r}/r$

(D) $\text{div}\,\vec{r} = 3$　(E) $\text{div}\,\vec{r} = 3r$

12. (5pt) Find $\text{div}\frac{\vec{r}}{r^3}$ $(r \neq 0)$

(A) $\text{div}\,\vec{r}/r^3 = 0$　(B) $\text{div}\,\vec{r}/r^3 = \vec{r}$　(C) $\text{div}\,\vec{r}/r^3 = \vec{r}/r$

(D) $\text{div}\,\vec{r}/r^3 = \vec{r}/r^2$　(E) $\text{div}\,\vec{r}/r^3 = \vec{r}/r^3$

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13. (5pt) In the polar coordinates, the gradient of a function $f(r,\theta,\phi)$ is given by

(A) $\text{grad}\, f = \frac{\partial f}{\partial r} + \frac{\partial f}{\partial \theta} + \frac{\partial f}{\partial \phi}$

(B) $\text{grad}\, f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{\partial f}{\partial \phi}\vec{e}_\phi$

(C) $\text{grad}\, f = \frac{1}{r}\frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{\sin\theta}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{\cos\phi}\frac{\partial f}{\partial \phi}\vec{e}_\phi$

(D) $\text{grad}\, f = \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\vec{e}_\phi$

(E) $\text{grad}\, f = \frac{1}{r}\frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{\sin\theta}\frac{\partial f}{\partial \theta}\vec{e}_\theta + \frac{1}{r\cos\phi}\frac{\partial f}{\partial \phi}\vec{e}_\phi$

14. (5pt) The Dirac delta function can be expressed by

(A) $\delta(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ikx}dk$　(B) $\delta(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ikx}dx$

(C) $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikx}dk$　(D) $\delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ikx}dx$

(E) $\delta(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ikx}dk$

15. (5pt) The Fourier transform of a function $f(x)$ is defined by

$$F(\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-i\xi x}dx$$

The inverse transform of $F(\xi)$ is equal to $f(x)$, if the inverse transform is defined by

(A) $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\xi)e^{i\xi x}dx$　(B) $f(x) = \int_{-\infty}^{\infty} F(\xi)e^{i\xi x}d\xi$

(C) $f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(\xi)e^{-i\xi x}d\xi$　(D) $f(x) = \int_{-\infty}^{\infty} F(\xi)e^{-i\xi x}dx$

(E) $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\xi)e^{-i\xi x}dx$

16. (5pt) The Fourier transform of $\frac{df(x)}{dx}$ is found to be

(A) $\frac{dF(\xi)}{dx}$,　(B) $\frac{dF(\xi)}{d\xi}$,　(C) $i\xi F(\xi)$,　(D) $2\pi i\xi F(\xi)$,　(E) $\int_{-\infty}^{\infty} F(\xi)d\xi$

17. (5pt) The Fourier transform of e^{iax} is found to be

(A) $2\sqrt{\pi}\delta(\xi)$　(B) $\delta(\xi - a)$　(C) $\frac{1}{a}\delta(\xi - a)$　(D) $\frac{1}{\sqrt{2\pi}}\delta(\xi + a)$　(E) $\frac{d}{d\xi}\delta(\xi)$

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18. (5pt) The Fourier transform of e^{-x^2} is found to be

(A) $2\pi\delta(\xi)$ (B) $\frac{1}{2\sqrt{\pi}}e^{-\frac{\xi^2}{4}}$ (C) $\frac{1}{\sqrt{2\pi}}e^{-\frac{\xi^2}{2}}$ (D) $\frac{1}{\sqrt{2\pi}}\delta(\xi)$ (E) $\frac{d}{d\xi}\delta(\xi)$

19. (5pt) Find the local maximum of a function $f(x) = 3x^3 - 4x + 1$

(A) $f(x_M) = \frac{36}{9}$ at $x_M = -\frac{2}{5}$

(B) $f(x_M) = \frac{25}{4}$ at $x_M = -\frac{2}{3}$

(C) $f(x_M) = \frac{25}{9}$ at $x_M = -\frac{2}{3}$

(D) $f(x_M) = \frac{2}{3}$ at $x_M = -\frac{2}{5}$

(E) $f(x_M) = \frac{5}{9}$ at $x_M = \frac{1}{3}$

20. (5pt) Find the maximum of the function $f(x,y) = 2x + 3y$ under the constraint $x^2 + y^2 = 1$.

(A) $f(x_M, y_M) = \sqrt{7}$ at $(x_M, y_M) = \left(-\frac{2}{\sqrt{5}}, \frac{7}{\sqrt{5}}\right)$

(B) $f(x_M, y_M) = \sqrt{9}$ at $(x_M, y_M) = \left(-\frac{2}{\sqrt{5}}, -\frac{3}{\sqrt{5}}\right)$

(C) $f(x_M, y_M) = \sqrt{11}$ at $(x_M, y_M) = \left(-\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}\right)$

(D) $f(x_M, y_M) = \sqrt{13}$ at $(x_M, y_M) = \left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)$

(E) $f(x_M, y_M) = \sqrt{15}$ at $(x_M, y_M) = \left(\frac{2}{\sqrt{15}}, \frac{3}{\sqrt{15}}\right)$

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