

國立成功大學

113學年度碩士班招生考試試題

編 號：99

系 所：土木工程學系

科 目：工程數學

日 期：0201

節 次：第 3 節

備 註：可使用計算機

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (25%) Consider the following second-order linear nonhomogeneous differential equation with constant coefficients: $y'' + 4y' - 5y = 10te^{-t}$,

(a) Find the solution to the corresponding homogeneous equation. (5%)

(b) Find a particular solution to the nonhomogeneous equation. (15%)

(c) State the general solution to the differential equation. (5%)

2. (25%) Given the matrix: $A = \begin{bmatrix} 8 & 3 \\ -3 & 2 \end{bmatrix}$, $n=9$. Perform the following tasks:

(a) Determine whether the matrix A has an eigenvalue λ_1 of multiplicity two. (6%)

(b) Assuming λ_1 is an eigenvalue of multiplicity two, explain why the equation $\lambda^n = c_0 + c_1\lambda$ does not yield enough independent equations to solve for the coefficients c_i . (6%)

(c) Show how to use the derivative to of the eigenvector equation evaluated at λ_1 to obtain an extra equation needed to form a complete system. (6%)

(d) Compute A^n and use this result to compute the indicated power of the matrix A . (7%)

3. (20%) Given the scalar field $g = e^{x^2+y^2+z^2}$ and the vector field $\mathbf{v} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$, find the following:

(a) $\text{div}(g\mathbf{v})$ (4%)

(b) $\nabla^2 g$ (4%)

(c) $\text{curl}(\text{grad } g)$ (4%)

(d) $\text{div}(\text{curl } \mathbf{v})$ (4%)

(e) $\text{grad}(\mathbf{v} \cdot \mathbf{v})$ (4%)

4. (20%) Find the Fourier integral representation of the piecewise-continuous function

$$f(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 < x < 3 \\ 0 & x > 3 \end{cases}$$

5. (10%) Find an LU-factorization of

$$\begin{bmatrix} 3 & 3 & -6 \\ 2 & -2 & 4 \\ 1 & 1 & -2 \end{bmatrix}$$