國立成功大學

113學年度碩士班招生考試試題

編　號：108

系　所：工程科學系

科　目：訊號與系統

日　期：0202

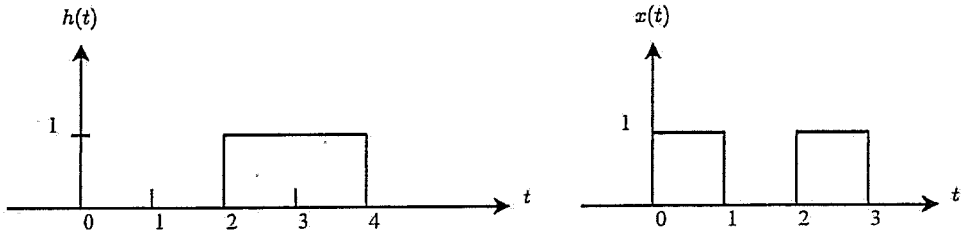
節　次：第 2 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1) **Problem 1 (20%):**

a) (10%) An LTI system with its impulse response $h(t)$ and its input signal $x(t)$ is shown below. Use the convolution integral to determine and plot its output signal $y(t)$.



b) (10%) An LTI system has its impulse response as $h(t) = 2\text{sinc}^2(2t)$. If $x(t) = \text{sinc}(t)$ is its input signal, what is its output signal $y(t)$ and its Fourier transform $Y(f)$, respectively? (Hint: $\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$)

2) **Problem 2 (10%):**

A. Determine whether each of the following systems is **linear or non-linear**, and whether each of the following systems is **time-invariant or time-varying**.

a) (3%) $y(t) = x^2(t) + e^{-|x(0.5-t)|}$

b) (3%) $y(t) = t^2 x(2-t)$

B. Determine whether each of the following systems is **causal or non-causal**.

a) (2%) $h(t) = e^t u_{-1}(t)$

b) (2%) $y[n] = x[n-2] + x[n-1] + x[n] + x[n+1] + x[n+2]$

3) **Problem 3 (10%):**

a) (5%) Calculate sinc(x)*sinc($2x$), where "*" denotes convolution operation.

b) (5%) Find the impulse response of a system with its frequency response as

$$H(f) = \frac{\sin(2\pi f)\sin^2(6\pi f)}{(2\pi f)^2}.$$

4) **Problem 4 (10%):**

a) (4%) Determine if the signal below is an energy-type or power-type signal.

$$x(t) = e^{-5t},\ t \geq 0.$$

b) (3%) What is its power (or energy)?

c) (3%) What is its power spectral density (or energy spectral density)?

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

5) **Problem 5 (20%):**

Use Parseval's relation and the properties of the Fourier series to solve the following problems.

a) (10%) A periodic signal $x(t)$ is real with a period $T = 8$. The non-zero Fourier series coefficients for $x(t)$ are specified as

$$a_1 = a_{-1}^* = j,$$

$$a_5 = a_{-5} = 2.$$

Express signal $x(t)$ in the form of sinusoidal signals.

b) (10%) Assume a truncated signal is $y(t) = \Pi(t)x(t)$, where $\Pi(t)$ denotes a rectangular function. Compute power spectral density (PSD) function $S_y(f)$ of $y(t)$, where $x(t) = \cos(2\pi f_0 t + \theta)$, and f_0 and θ are constants. Hint: $\delta(f)$ can be derived from a limit as follows

$$\delta(f) = \lim_{T\to\infty} \frac{1}{T}\Big(\frac{\sin(\pi T f)}{\pi f}\Big)^2.$$

6) **Problem 6 (20%):**

Use the properties of Fourier transform to solve the following problems.

a) (10%) We have the following Fourier transform pairs as

$$t\dot{x}(t) \iff j\frac{d}{d\omega}X(\omega),$$

$$x(t) = e^{-at}u_{-1}(t) \iff X(\omega) = \frac{1}{a+j\omega},\ a > 0,$$

where $u_{-1}(t)$ is the unit step function and $\omega = 2\pi f$. Find the inverse Fourier transform of

$$X(\omega) = \frac{1}{(2+j\omega)^3}.$$

b) (10%) Given an LTI system and a signal $x(t) = 2e^{-3t}u_{-1}(t-1)$, determine the impulse response $h(t)$ of the system, where its input is $\frac{d}{dt}x(t)$ and its output is $-3y(t) + e^{-2t}u(t)$. You may also need the following Fourier transform pair:

$$\frac{d}{dt}x(t) \iff j\omega X(\omega).$$

7) **Problem 7 (10%):**

If a periodic signal $x(t)$ with its period T is $x(t) = \sum_{k=-\infty}^{\infty} p(t - nT)$, and a rectangular pulse signal $p(t)$ is

$$p(t) = \begin{cases} 1, & -\frac{T}{4} < t < \frac{T}{4}, \\ 0, & \text{otherwise.} \end{cases}$$

Determine the Fourier transform of $x(t)$.