

國立成功大學

113學年度碩士班招生考試試題

編 號： 71

系 所： 機械工程學系

科 目： 工程數學

日 期： 0201

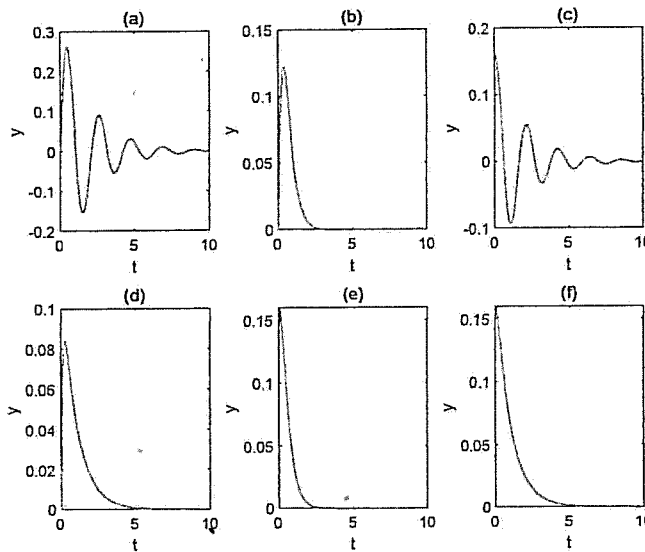
節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Given that $f(t) = \begin{cases} 0.16, & t < 0 \\ 0, & t \geq 0 \end{cases}$ is a Heaviside step function and $y(t)$ is the output, which of the following figures can be generated from $y''(t) + y'(t) + 9 = f(t)$? (10%)
 ((Select all that apply). Justify your answers.

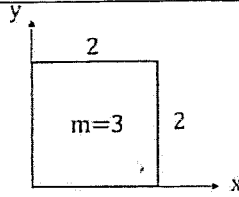
Note: no points will be given if the explanation is incorrect



2. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ if $\mathbf{F}(x, y) = e^y \mathbf{i} - \sin(\pi x) \mathbf{j}$ and C is the triangle with vertices $(1,0), (0,1), (-1,0)$ traversed counterclockwise. (20%)
3. The rotational kinetic energy $T_{rot} = \frac{1}{2} I \omega^2$ can be represented in matrix form as, (20%)

$$T_{rot} = \frac{1}{2} (\omega_x \quad \omega_y \quad \omega_z) \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

where I_{xx}, I_{yy}, I_{zz} are moments of inertia with respect to the x, y and z axis, respectively. $I_{xy}, I_{xz}, I_{yx}, I_{yz}, I_{zx},$ and I_{zy} are products of inertia. A square flat plate has mass (m) of 3 and side length of 2. The rotation kinetic energy of the rotation around the axis x' with the x, y, z coordinate axes, and the origin can be represented as:



$$T_{rot} = \frac{1}{2} (\omega_x \quad \omega_y \quad \omega_z) \begin{pmatrix} 4 & -3 & 0 \\ -3 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

- (i) Find the direction of new coordinate x' , y' and z' , called principal axes, in which the object can rotate around at a constant angular speed without a need for any torque. In other words,

$$T_{rot} = \frac{1}{2} (\omega_{x'} \quad \omega_{y'} \quad \omega_{z'}) \begin{pmatrix} I_{x'x'} & 0 & 0 \\ 0 & I_{y'y'} & 0 \\ 0 & 0 & I_{z'z'} \end{pmatrix} \begin{pmatrix} \omega_{x'} \\ \omega_{y'} \\ \omega_{z'} \end{pmatrix}$$

- (ii) Find the corresponding $I_{x'x'}$, $I_{y'y'}$, and $I_{z'z'}$.

4. To solve the PDE by Laplace transforms

$$x \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} = xt, w(x, 0) = 0 \text{ if } x \geq 0, w(0, t) = 0 \text{ if } t \geq 0 \quad (15\%)$$

5. To find the complex Fourier integral of the $f(x) = e^{-|x|}$, $-\infty < x < \infty$ (10%)

6. To prove that

(a) The Fourier sine series $\frac{\pi-x}{2} = \sum_{n=1}^{\infty} \frac{\sin nx}{n}$; $(0 \leq x \leq \pi)$, (8%)

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (7%)

7. Find the Laplace transformation of the following function. (10%)

