國立成功大學

113學年度碩士班招生考試試題

編　號：324

系　所：經濟學系

科　目：統計學

日　期：0201

節　次：第 1 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

※ **Instructions: No point will be given if there is no explanation, no calculation, or your answers aren't understandable.**

1. [20 pts] Consider the following model:

$$\log(\text{crime}_i) = \alpha + \beta \log(\text{population}_i) + \varepsilon_i,$$

which relates the annual number of crimes of a city to population.

(a) [5 pts] What is the expected sign of β? How do we interpret β?

(b) [5 pts] Suppose we want to test H_0: $\beta \leq 1$ vs. H_1: $\beta > 1$ at 5% significance level. Using the following information to test the hypothesis:

$$\widehat{\log(\text{crime}_i)} = \underset{}{-6.6} + \underset{(1.05)}{1.5} \log(\text{population}_i), \quad \underset{(0.12)}{}$$

where numbers in parentheses are standard errors.

(c) [10 pts] Can we use the result mentioned above to claim that there is a causal effect of population on crime? If yes, provide an explanation. If not, provide an assumption to make it a valid claim.

2. [30 pts] Let the variable Y_i be generated by $Y_i = X_i^2 + \varepsilon_i$, where ε_i is independent of X_i, $E(\varepsilon_i) = 0$, and $E(\varepsilon_i^2) = \sigma^2$. It is further assumed that $E(X_i) = 0$. Now suppose that Dr. Strange would like to estimate the following regression model:

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

(a) [10 pts] Find expressions for β_0 and β_1 in terms of the moments of X_i.

(b) [10 pts] Propose analog estimators for β_0 and β_1, denoted by $\hat{\beta}_0$ and $\hat{\beta}_1$.

(c) [10 pts] Are $\hat{\beta}_0$ and $\hat{\beta}_1$ consistent estimators? Be clear about the theorems you use.

3. [20 pts] Let $U = \max(X, Y, Z)$, and $W = \min(X, Y, Z)$, where $(X, Y, Z) \sim^{i.i.d.} U[a, b]$.

   (a) [5 pts] Find the p.d.f. of U.

   (b) [5 pts] Find the p.d.f. of W.

   (c) [10 pts] Find the joint p.d.f. of U and W.

4. [20 pts] Assume you possess a chopstick initially measuring one unit in length. Subsequently, an individual arrives and fractures a segment at a point selected uniformly at random. This results in a remaining chopstick of length Y. The residual piece undergoes another break at a randomly chosen point, uniformly distributed over the remaining portion, leaving you with a chopstick of length X. Finally, the remaining piece is once again broken at a random point, uniformly distributed over the remaining portion, and you are left with a chopstick of length Z.

   (a) [10 pts] Find the pdf of Z.

   (b) [5 pts] Find $E(Z)$.

   (c) [5 pts] Find $Var(X)$.

5. [10 pts] Researchers want to know if a subject has ESP (extra-sensory perception). They set up a test that consists of 6 trials. In each trial, a coin is flipped, and the subject guesses the side of the coin. The hypotheses are:

   H_0 : the subject does not have ESP.

   H_1 : the subject is more likely to guess the side.

   Suppose that the subject is successfully for 4 out of 6. What is the exact p-value (not an approximation) for this test?

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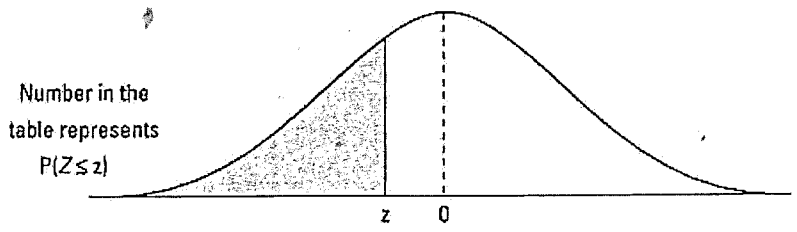

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
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−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
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−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
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−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
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−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641