國立成功大學

113學年度碩士班招生考試試題

編　號：36

系　所：數學系應用數學

科　目：高等微積分

日　期：0202

節　次：第 2 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Evaluate the surface integral.

$$\iint_S x^2 dS, \quad \text{where } S \text{ is the sphere } (x-1)^2 + y^2 + z^2 = 1.$$

2. (15%) Suppose that $D \subset \mathbb{R}^3$ and its boundary ∂D is a piecewise smooth C^1 surface oriented positively with unit normal vector n. Show that

$$\iiint_D (u\Delta v - v\Delta u)dV = \iint_{\partial D} (u\nabla v - v\nabla u) \cdot n dS$$

for all C^2 functions $u, v : D \to \mathbb{R}$.

3. (15%) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a C^2 function. Prove that if f has a local minimum at $x = a$, then $f'(a) = 0$ and $f''(a) \geq 0$.

4. (15%) Let f be a real valued function on a closed interval $[a, b]$. Prove that if f is continuous on $[a, b]$, then f attains its absolute maximum and absolute minimum in $[a, b]$.

5. (45%) Suppose that $\{f_n\}$ is a sequence of real functions defined on $[a, b]$ and that $f_n \to f$ pointwise on $[a, b]$. Determine whether the following statements are true or false. Prove the statement if it is true, and give a counterexample if it is false.

   a. (15%) If f_n is differentiable on (a, b) for each $n \in \mathbb{N}$ and $f_n \to f$ uniformly on $[a, b]$, then f is differentiable on (a, b) and

   $$f'(x) = \lim_{n\to\infty} f_n'(x) \quad \text{for } x \in (a, b).$$

   b. (15%) If f_n is Riemann integrable on $[a, b]$ for each $n \in \mathbb{N}$ and $f_n \to f$ uniformly on $[a, b]$, then f is Riemann integrable on $[a, b]$ and

   $$\int_a^b f(x)dx = \lim_{n\to\infty} \int_a^b f_n(x)dx.$$

   c. (15%) If f_n is continuous and uniformly bounded on $[a, b]$ for each $n \in \mathbb{N}$, then $\{f_n\}$ is equicontinuous on $[a, b]$.