國立成功大學

113學年度碩士班招生考試試題

編　號：197

系　所：電機資訊學院-資訊聯招

科　目：程式設計

日　期：0201

節　次：第 2 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part I. 資料結構 (50%)

注意: 資料結構共有 10 大題，每大題 5 分，共 50 分。請在答案卷第一頁做表如下，將您的答案整理於該表中。否則，不予計分。

1. i)	1. ii)	2. i)	2. ii)	2. iii)	3. i)	3. ii)	4. i)
4. ii)	5. i)	5. ii)	6.	7.	8.	9.	10.

1. **[3%, 2%]** You are given an empty binary search tree (BST).
   i) [Step 1] Please successively insert the data pairs containing the following keys 15, 8, 13, 18, 17, 6, 11, 14, 5 into the tree. What is the level of the node containing 5 in the resultant BST?
   Note: In a tree, each step from top to bottom is called as level of a tree. The level count starts with 1 and increments by 1 at each level or step.
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) None of the above
   ii) [Step 2] After Step 1 is executed, please delete the node containing 15 from the BST. To delete a nonleaf node, the pair to be deleted should be replaced by the largest pair in its left subtree or the smallest one in the right subtree. Which of the following keys is a suitable key to replace 15?
   (A) 8
   (B) 13
   (C) 17
   (D) 18
   (E) None of the above
2. **[1%, 1%, 3%]** Considering the data pairs with keys in the given order: 20, 5, 10, 18, 4, 22, 11, 32, 21, as inputs to create a tree.
   i) If you create a max heap for them, what is the level of the node containing 5?
   (A) 1
   (B) 2
   (C) 3
   (D) 4

(E) None of the above

ii) If you create a min heap for them, which of the following statements is true?

(A) The node containing 11 is in the left subtree of the root.

(B) The node containing 18 is in the right subtree of the root.

(C) The node containing 20 is the root.

(D) The node containing 21 is at level 3.

(E) None of the above.

iii)If you create a symmetric min-max heap for them, which of the following statements is true?

(A) To delete the node containing 5, the node containing 21 is used to replace the deleted node.

(B) There are two nodes at level 4.

(C) The leftmost grandchild of the root is 5.

(D) The keys of the nodes in the right subtree of the root are 32, 10, and 20.

(E) None of the above.

3. **[2%, 3%]** Given a binary tree with the following postfix and infix patterns.

postfix: Z, P, C, L, Y, K, S, W, R, Q

infix: Z, Y, P, L, C, Q, K, R, W, S

i) What is the result of level-order traversal for the tree?

(A) C, S, R, Z, L, K, W, Q, Y, P

(B) Q, Y, R, Z, L, K, W, P, C, S

(C) Q, R, Y, W, K, L, Z, S, C, P

(D) Y, K, R, Z, Q, L, W, C, P, S

(E) None of the above

ii) What is the result of preorder traversal for the tree?

(A) Q, Y, Z, L, P, C, R, K, W, S

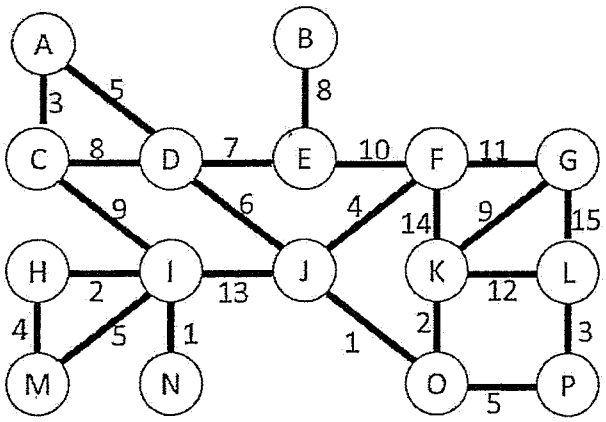
(B) P, Z, C, Y, L, Q, K, R, W, S

(C) Q, Y, Z, L, P, C, R, K, S, W

(D) P, Z, C, Y, L, Q, K, R, S, W

(E) None of the above

4. **[2%, 3%]** Given the following weighted undirected graph.



i) Which of the following statements is correct?

(A) When performing a depth first search of this graph, the next node to visit can be D after visiting A, C, I, N, M, and H.

(B) When performing a breath first search of this graph, the next node to visit can be D after visiting A, C, I, N, M, and H.

(C) When performing a depth first search of this graph, the next node to visit can be K after visiting B, E, and F.

(D) When performing a breath first search of this graph, the next node to visit can be K after visiting B, E, and F.

(E) None of the above

ii) After generating a maximum cost spanning tree from this graph, which of the following statements is correct?

(A) The degree of the node F in the resultant maximum cost spanning tree is 3.

(B) The cost of the resultant maximum cost spanning tree is 118.

(C) (O, P) is an edge of the resultant maximum cost spanning tree.

(D) In the resultant maximum cost spanning tree, the number of edges on the path from A to O is 6.

(E) None of the above

5. **[3%, 2%]** Given the following bloom filter.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
1	1	0	1	1	1	1	1	0	1	0	0	0	1	0

i) Assume that the bloom filter has 3 hash functions $f_1(k)$, $f_2(k)$, and $f_3(k)$:

$$f_1(k) = (3k) \bmod m,$$

$$f_2(k) = (2k+1) \bmod m,$$

$$f_3(k) = k^2 \bmod m,$$

where k represents the key and m is the size of bit array for the bloom filter. Which of the following statements is true?

(A) When inserting k=12, the bits in the bloom filter are not changed.

(B) To delete k=2, the bits at indices 4, 5, and 6 are changed to 0.

(C) When querying the bloom filter for k=15, the answer is "yes."

(D) When querying the bloom filter for k=8, the answer is "yes."

(E) None of the above.

ii) Considering that the elements in the bloom filter is 5. To minimize the false positive probability, what is the ideal optimum number of hash functions?

(A) 15

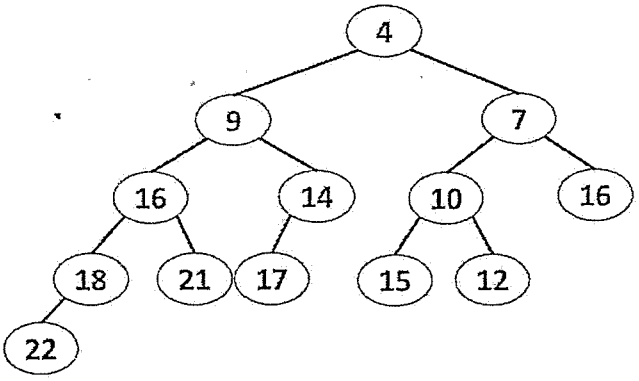
(B) 5

(C) 3*ln(2)

(D) 5*ln(2)

(E) None of the above

6. **[5%]** Given the following height-based leftist tree.



After deleting the minimum element from the leftist tree, which of the following statement is true?

(A) The number of nodes in the right subtree of the root is 7.

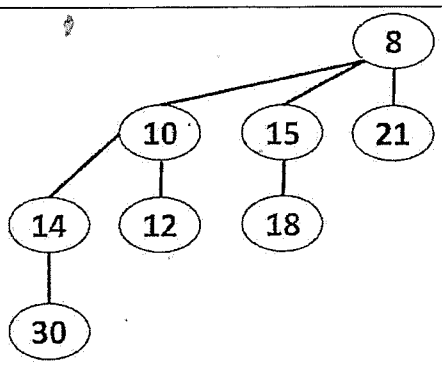
(B) The node containing 15 is at level 3.

(C) The shortest path from the root to the external node contains 2 edges.

(D) The node containing 16 is in the right subtree of the root.

(E) None of the above

7. **[5%]** Given an initial empty Fibonacci heap (F heap). After a sequence of insertion operations and delete-min operations, the Fibonacci heap becomes as follows:



Then, please proceed to perform operations in the following order: decrease the key 14 by 5, decrease the key 21 by 14, delete the key 12, insert the key 14, delete the minimum key. Considering these operations, which of the following statements is true?

(A) After performing "delete the key 12", the F heap has two min trees.

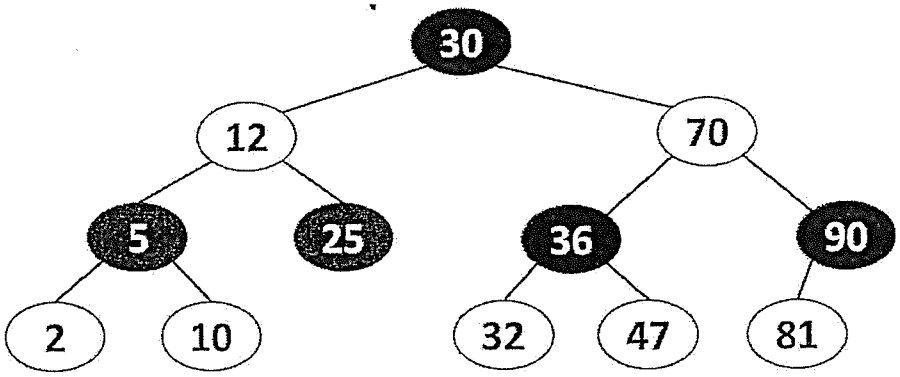
(B) A cascading cut occurs when performing "decrease the key 14 by 5."

(C) The resultant F heap has 4 min trees.

(D) The resultant F heap has 2 min trees.

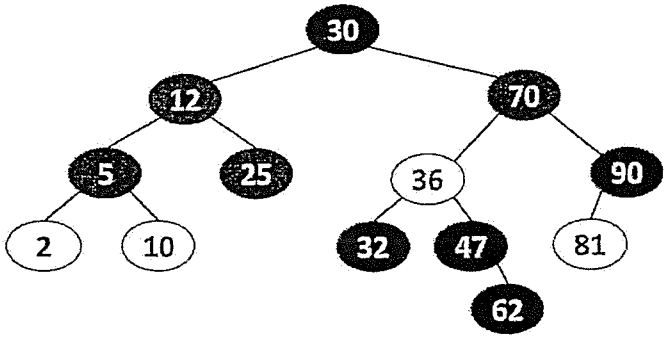
(E) None of the above.

8. [5%] Given the following red-black tree, where the dark color indicates a black node and white color indicates a red node.

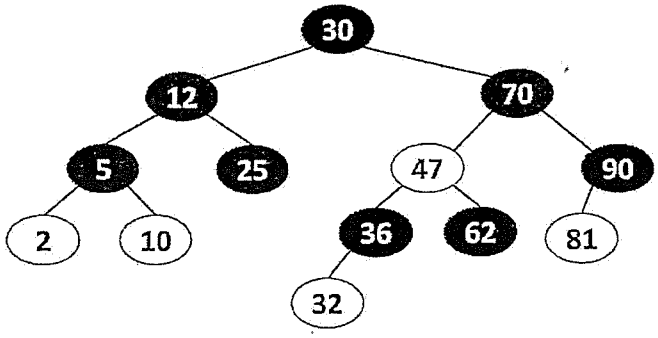


After inserting 62, which of the followings is true?

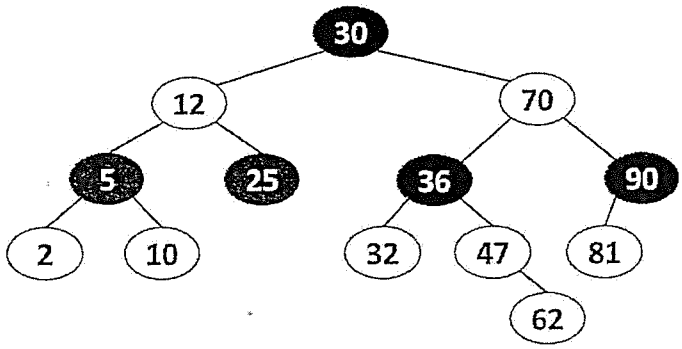
(A)



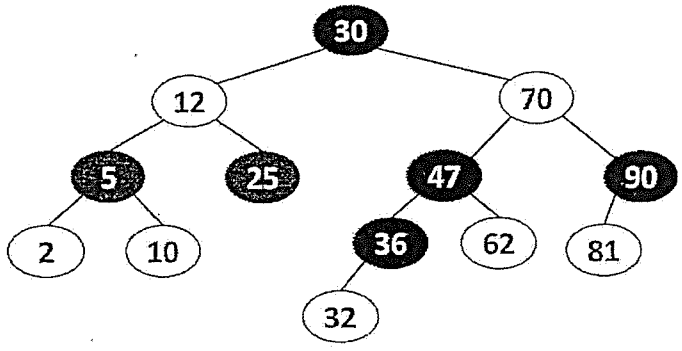
(B)



(C)

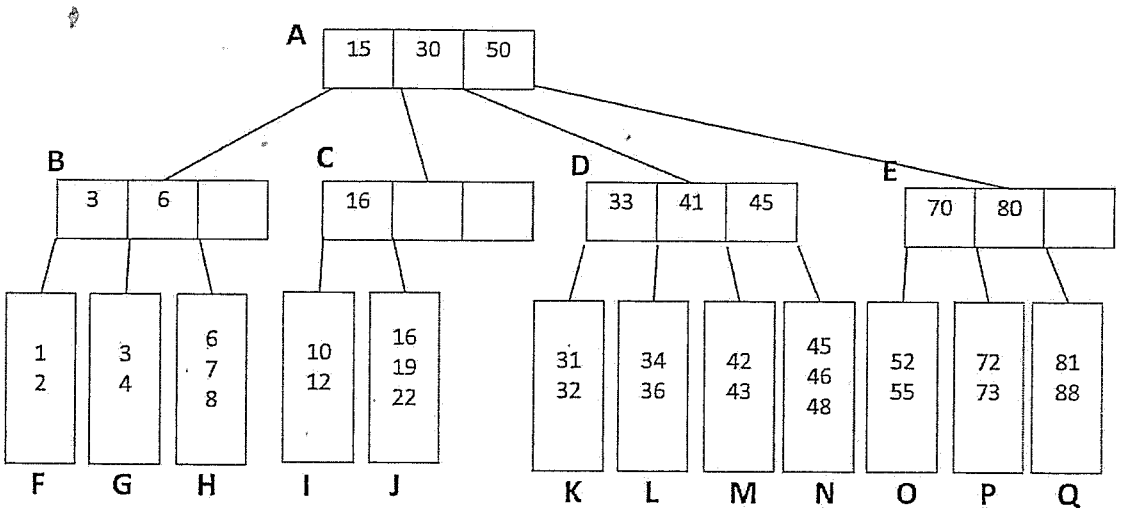


(D)



(E) None of the above.

9. [5%] Given the following B^+-tree of order 4 (2-3-4 tree). The capacity of a data node is 3 and a data node has at least two elements.



Which of the following statement is true?

(A) After inserting the element whose key is 49 into the B^+-tree, 45 becomes in the root.

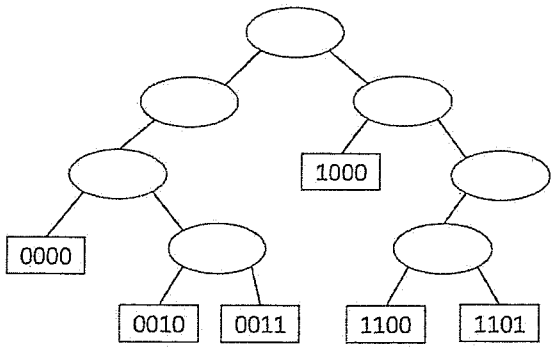
(B) After deleting 72 from the B^+-tree, the number of keys in node E is 2.

(C) After inserting the element whose key is 28 into the B^+-tree, the key in node C is 16.

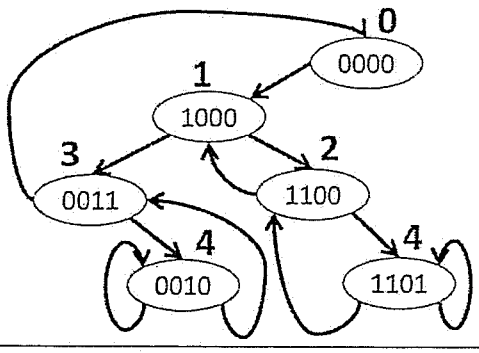
(D) After deleting 45 from the B^+-tree, the keys in node D are 33, 41, and 45.

(E) None of the above

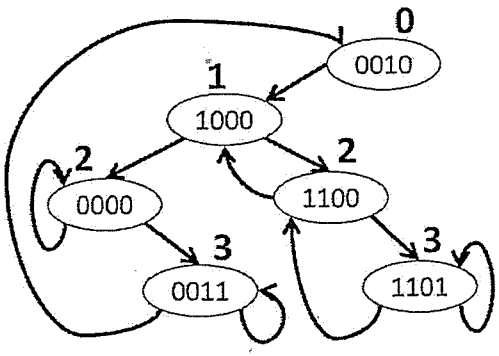
10. **[5%]** A binary trie can be transformed to Patricia. Considering the following binary trie, which of the Patricia is a correct transformation result?



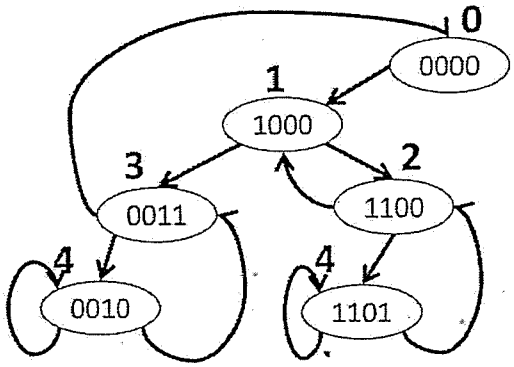
(A)



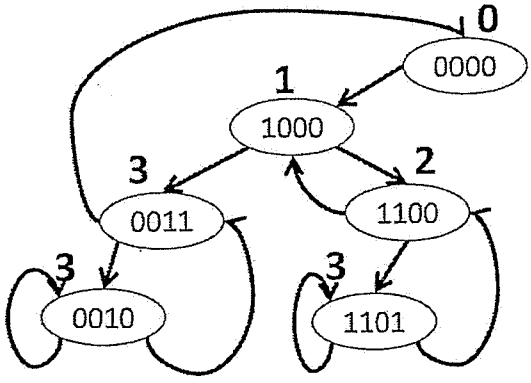
(B)



(C)



(D)



(E) None of the above

Part II. 演算法 (50%)

11. (10%) Consider three $n \times n$ matrices, denoted as A, B, and C. Each of these matrices is partitioned into four $n/2 \times n/2$ submatrices. Assuming that n is an exact power of 2, we can guarantee that, for $n \geq 2$, the dimension $n/2$ is an integer. The subdivisions for matrix A yield A_{11}, A_{12}, A_{21}, and A_{22}. Similarly, matrix B is decomposed into B_{11}, B_{12}, B_{21}, and B_{22}, while matrix C undergoes division into C_{11}, C_{12}, C_{21}, and C_{22}. We have the following procedure:

```
F(A, B)
    n = A.rows
    if n == 1
        c11 = a11 · b11
    else
        C11 = F(A11, B11)+F(A12, B21)
        C12 = F(A11, B12)+F(A12, B22)
        C21 = F(A21, B11)+F(A22, B21)
        C22 = F(A21, B12)
    return C
```

It is important to note that the procedure for C_{22} is invoked only once and '+' is matrix addition. Let $T(n)$ represent the time required to process two $n \times n$ matrices using this particular procedure. Provide an asymptotic tight bound (Θ) for $T(n)$, assuming that $T(n)$ is a constant for sufficiently small n.

12. (10%) Consider a neural network with n layers of fully connected layers, where the first layer is the input layer, the nth layer is the output layer, and the rest are hidden layers. The ith layer has P_i neurons, where $1 \leq i \leq n$. Each neuron in the ith layer is connected to every neuron in the $(i+1)$th layer, where $1 \leq i < n$. Let $w_{j,k}^i$ represent the weight of the connection from the jth neuron in the ith layer to the kth neuron in the $(i+1)$th layer. Let $[v_1^i \; v_2^i \; \cdots \; v_{P_i}^i]^T$ be the values of neurons in the ith layer, where $1 \leq i \leq n$. Then, we can obtain $v_k^{i+1} = \sum_{j=1}^{P_i} w_{j,k}^i v_j^i$, where $1 \leq k \leq P_{i+1}$. Given that $\langle P_1, P_2, P_3, P_4, P_5, P_6 \rangle = \langle 10, 6, 12, 5, 50, 3 \rangle$, we want to achieve the result from the input layer to the output layer in the fastest possible way. Please determine the minimum number of multiplications required for this neural network.

13. (10%) Given two sequences $X = \langle x_1, x_2, \ldots, x_m \rangle$ and $Y = \langle y_1, y_2, \ldots, y_n \rangle$, define $c[i,j]$ to be the length of an LCS (longest common subsequence) of the sequences $X_i = \langle x_1, x_2, \ldots, x_i \rangle$ and $Y_j = \langle y_1, y_2, \ldots, y_j \rangle$. Write the recursive formula to compute $c[i,j]$.

14. (10%) Given a sequence of n numbers, what is the lower bound for sorting algorithms employing comparison and exchange operations?
15. (10%) Find the set of feasible solutions $\{x = (x_1, x_2, x_3, x_4, x_5)\}$ or determine that no feasible solution exists for the following system of difference constraints:

$$x_1 - x_3 \leq 1$$
$$x_2 - x_3 \leq 4$$
$$x_4 - x_5 \leq -2$$
$$x_3 - x_4 \leq 4$$
$$x_5 - x_1 \leq 3$$
$$x_4 - x_2 \leq -7$$
$$x_1 - x_2 \leq -2$$
$$x_5 - x_3 \leq 1$$