

國立成功大學

113學年度碩士班招生考試試題

編 號：180

系 所：電腦與通信工程研究所

科 目：通訊系統

日 期：0201

節 次：第 2 節

備 註：可使用計算機

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Consider a linear time-invariant (LTI) system whose unit impulse response is $h(t)$. Answer the following questions based on the information:

$$p(t) * h(t) = q(t)$$

where $p(t)$ and $q(t)$ are both signals, and $*$ represents the convolution. (You can assume that $p(t)$, $h(t)$, and $q(t)$ are known.)

- (a) Find $p(t-T) * h(t)$ where $T > 0$ is a constant. [5 points]
 (b) Justify your answer to part (a) based on intuition (直觀) without using any equations. [5 points]
 (c) Find $p(t-T) * h(t-T)$ where $T > 0$ is a constant. [5 points]
2. (35%) Suppose that a white Gaussian noise (WGN) process $n(t)$ with two-sided power spectral density (PSD) $S_n(f) = \frac{N_0}{2}$ is passed through an LTI system with unit impulse response $g(t)$ where $g(t) = 1$ if $0 < t < 2$ and $g(t) = 0$ elsewhere. Denote the resulting output as $y(t)$. Answer the following questions.
- (a) Find the PSD of $y(t)$. [10 points]
 (b) Is $y(t)$ a strict-sense stationary process? Justify your answer. [10 points]
 (c) Are $y(100)$ and $y(102)$ independent random variables? Explain why. [10 points]
 (d) Justify your answer to part (c) based on the properties of WGN. [5 points]
3. (10%) Consider the transmission of a binary data sequence $\{a_n\}$, where the a_n take on values from the set $\{-1, +1\}$ with equal probability, using binary antipodal signaling. The overall pulse response is $p(t) = g(t) * c(t) * h(t)$, where $g(t)$ is the transmit filter, $c(t)$ is the channel, $h(t)$ is the receiver matched filter, and $*$ represents the convolution. The sampled pulse $p_k = p(kT)$ is such that:

$$p_k = \begin{cases} 1, & k = 0 \\ 0.5, & k = 1 \\ 0.5, & k = 2 \\ 0, & \text{otherwise} \end{cases}$$

The samples at the output of the receiver matched filter are

$$y_n = p_0 a_n + \sum_{k \neq n} a_k p_{n-k} + z_n$$

where the z_n are independent zero-mean Gaussian random variables with variance σ^2 . A decision on a_n is made by only using the observation y_n , i.e., no equalization is used.

- (a) What are the possible values for the intersymbol interference term and their probabilities? [4 points]
 (b) Derive an expression for the probability of a bit error in terms of $Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-\frac{\beta^2}{2}} d\beta$. [4 points]
 (c) Now suppose that a zero-forcing equalizer is used. Would the zero-forcing solution minimize the peak distortion after equalization? [2 points]

4. (15%) A one-dimensional additive noise channel, $y = x + n$, has uniform noise distribution

$$f_N(n) = \begin{cases} \frac{1}{L}, & \text{if } |n| \leq \frac{L}{2} \\ 0, & \text{if } |n| > \frac{L}{2} \end{cases}$$

where $L/2$ is the maximum noise magnitude. The input x has binary with equally likely input values $x = \pm 1$. The noise is independent of x .

- (a) Determine the threshold such that the receiver has minimum error probability. [3 points]
 - (b) For what value of L is BER $P_e < 10^{-6}$? [3 points]
 - (c) Find the SNR (function of L). [3 points]
 - (d) Find the minimum SNR that ensures "error-free" transmission. [3 points]
 - (e) Repeat part (d) if 4-level PAM is used instead. [3 points]
5. (15%) Suppose the following signal vectors are employed on an AWGN channel with a two-sided noise power spectral density of $N_0/2$ watts/Hz.

$$s_1 = (a, 2a, a)$$

$$s_2 = (a, 0, a)$$

$$s_3 = (-a, 2a, a)$$

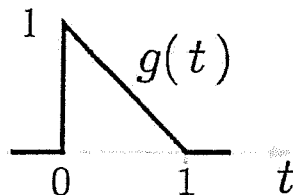
$$s_4 = (-a, 0, a)$$

- (a) Find the *exact* symbol error probability in terms of average modulated symbol energy-to-noise

ratio, using $Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-\frac{\beta^2}{2}} d\beta$. [5 points]

- (b) Suppose a new set of signal vectors is constructed by translating the vectors as follows: $\hat{s}_i = s_i - b$, $i = 1, \dots, 4$. What vector b will minimize the average symbol energy in the new signal constellation $\{\hat{s}_i\}_{i=1}^4$? [5 points]
- (c) How would the symbol error probability in part (b) compare with that in part (a)? [5 points]

6. (10%) A 64-PAM transmitter sends $s(t) = ag(t)$, where $a \in \{-63, -61, -59, \dots, 59, 61, 63\}$, and where $g(t)$ is sketched here:



Suppose the receiver observes the constant signal $r(t) = -18$ (for all t). Find the decision made by the minimum-distance receiver.