國立成功大學
113學年度碩士班招生考試試題

編　號：228

系　所：工業與資訊管理學系

科　目：作業研究

日　期：0202

節　次：第 2 節

備　註：可使用計算機

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1. (10%) Solve the following linear programming model using graphical solution method.

Maximize profit = 30X + 40Y

Subject to the constraints

Constraint 1: $4X + 3Y \le 12$

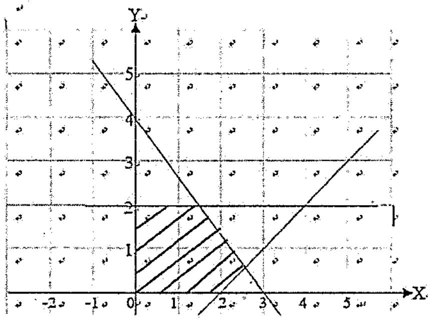
Constraint 2: $Y \le 2$

Constraint 3: $X - Y \le 2$

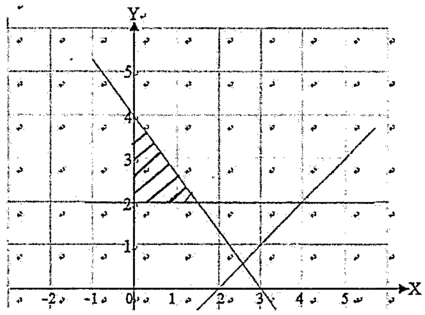
Constraint 4: $X, Y \ge 0$

1.1. Which shaded area in the graph is the feasible region for this problem? (5%)

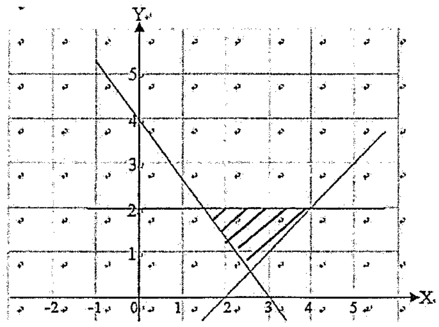
(a)



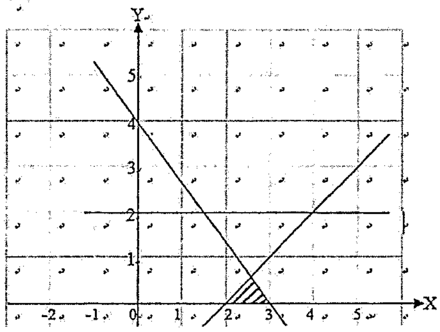
(b)



(c)



(d)



1.2. What is the optimal solution for this problem? (5%)

(a) $X = 0$, $Y = 2$

(b) $X = 1.5$, $Y = 2$

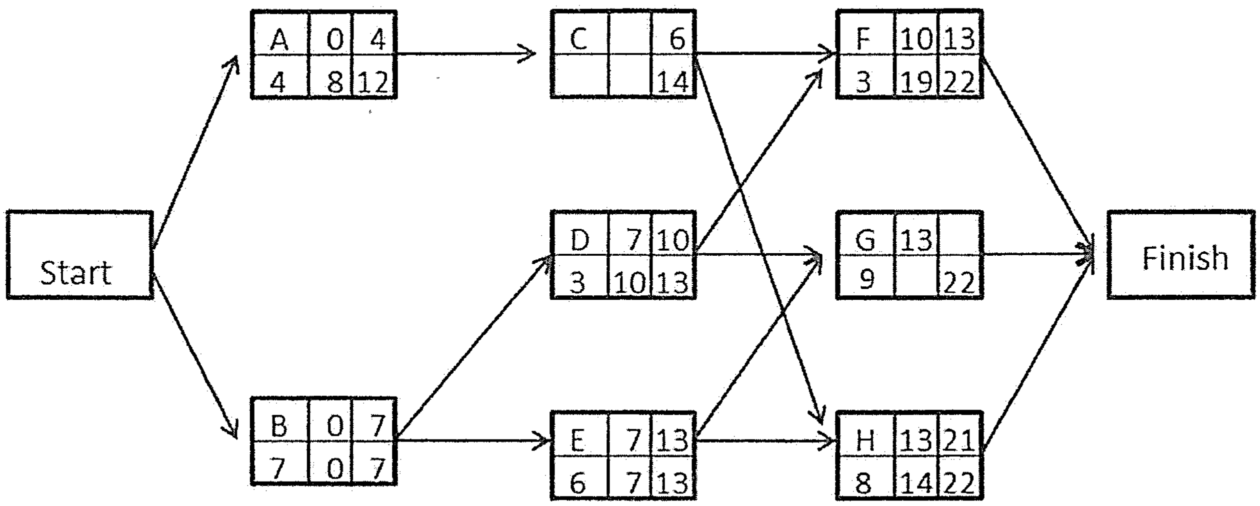
(c) $X = 2.0$, $Y = 0$

(d) $X = \frac{18}{7}$, $Y = \frac{4}{7}$

2. (20%) A project consists of 8 activities (Activity A to Activity H). The activity durations are known for sure. The project network is given with the following activity information:

Activity name	Earliest Start Time	Latest Start Time
Activity duration	Earliest Finish Time	Latest Finish Time

Use the project network to answer the questions 2.1 to 2.4.



2.1. What should be the correct numbers in the block for activity C? (5%)

(a)

C	3	6
3	11	14

(b)

C	12	6
2	12	14

(c)

C	4	6
2	12	14

(d)

C	5	6
1	13	14

2.2. What should be the correct numbers in the block for activity G? (5%)

(a)

G	13	22
9	11	22

(b)

G	13	22
9	12	22

(c)

G	13	22
9	13	22

(d)

G	13	22
9	14	22

2.3. Which path is the critical path? (5%)

(a) A → C → F
(b) A → C → H
(c) B → E → H
(d) B → E → G

Assume that all the given activity times have variance 0. The project manager needs to reduce the project completion time to 20 days. She has developed a linear programming (LP) model to help her make the crashing decisions. The decision variables are: T_i = start time for activity *i*, and C_i = the amount time by which activity *i*'s time is crashed.

2.4. Which of the following constraint is a necessary constraint in this LP model? **(5%)**

(a) $T_E + 7 - C_E \leq 13$ is a necessary constraint.

(b) $T_D + 3 \leq T_F + T_G$ is a necessary constraint.

(c) $T_H + 8 - C_H \leq 22$ is a necessary constraint.

(d) $T_H + 8 - C_H \leq 20$ is a necessary constraint.

3. (20%) In order to decide which decision alternative should be chosen, the **Profits** under each combination of a decision alternative and a state of nature is given in the following payoff table. The decision maker must choose one of the decision alternatives.

	State of nature 1	State of nature 2	State of nature 3
Prior probability of each state of nature	**0.2**	**0.5**	**0.3**
Decision alternative 1	-100	550	250
Decision alternative 2	-200	350	600
Decision alternative 3	-300	700	200
Decision alternative 4	-150	250	300

3.1. Which decision alternative should be chosen using the Maximin payoff criterion? (5%)

(a) Decision alternative 1

(b) Decision alternative 2

(c) Decision alternative 3

(d) Decision alternative 4

3.2. Which decision alternative should be chosen using the Maximum likelihood criterion? (5%)

(a) Decision alternative 1

(b) Decision alternative 2

(c) Decision alternative 3

(d) Decision alternative 4

3.3. Which decision alternative should be chosen using the Bayes' desicion rule? (5%)

(a) Decision alternative 1

(b) Decision alternative 2

(c) Decision alternative 3

(d) Decision alternative 4

3.4. Suppose it is possible to pay for the perfect information to know which state of nature will occur in the future, assuming the decision would be made based on the Bayes' decision rule, what is the difference between the expected profits with or without this information (i.e., the highest amount the decision maker would like to pay for this perfect information)? (5%)

(a) 100

(b) 160

(c) 180

(d) 220

4. (20 points) A simplified model for the spread of a disease goes this way: the total population size is $N = 5$, of which some are diseased and some are healthy. During any single period of time, two people are selected at random from the population and assumed to interact. The selection is such that an encounter between any pair of individuals in the population is just as likely as any other pair. If one of these persons is diseased and the other not, then with probability 0.1 the disease is transmitted to the healthy person. Otherwise, no disease transmission takes place.

Describe a Markov Chain model for the system. That is, specify the stochastic process X_n, the time index n, the state space, and the transition probability matrix.

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5. (30 points) Consider a (q, Q) policy, and it works as follows: if the stock level at the end of a period is **less than** q units, then additional Q units are ordered and will be available at the beginning of next period. Otherwise, no ordering takes place. Assume that demand that is not filled in a period is lost. Let D_n be the demand on period n and let X_n be the inventory level at the end of the nth period.

Suppose that $q = 2$, $Q = 2$, $X_0 = 3$, and D_1, D_2, ... are independent random variables with the following distribution:

k	$P(D_n = k)$
0	0.1
1	0.4
2	0.3
3	0.2

The state-space is {0,1,2,3} and the transition matrix **P** of $\{X_n\}$ is

$$\mathbf{P} = \begin{bmatrix} 0.5 & 0.4 & 0.1 & 0 \\ 0.2 & 0.3 & 0.4 & 0.1 \\ 0.5 & 0.4 & 0.1 & 0 \\ 0.2 & 0.3 & 0.4 & 0.1 \end{bmatrix}, \text{ and } \mathbf{M} = (\mathbf{I} - \mathbf{P}_{AA})^{-1} = \begin{bmatrix} 1.111 & 0 \\ 0.494 & 1.111 \end{bmatrix}$$

where the two rows correspond to state 2 and state 3. Any other matrix you need, you must calculate on your own.

a) (6 points) Show that $\{X_n\}$ is a Markov Chain.

b) (6 points) What is the probability that first re-order occurs after 2 periods? (i.e., $X_2 < q$ and $\{X_0, X_1\} \geq q$)

c) (6 points) What is the probability that a stockout occurs before a re-order (which is not a stockout) ?

d) (6 points) Compute the expected time until a re-order occurs.

e) (6 points) What is the probability that a stockout occurs at some point during the first 2 periods?