# 國立成功大學

# 113學年度碩士班招生考試試題

編　號：227

系　所：統計學系

科　目：統計學

日　期：0202

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

**A. True or False (Please answer "T" for true and "F" for false statement, 10 x 2% = 20%)**

1. Categorical data must be non-numeric.
2. If the mean of a normal distribution is negative, then the median must also be negative.
3. The probability of an intersection of two events is computed using the addition law.
4. The covariance of two variables X and Y must be between –1 and +1.
5. If arrivals follow a Poisson probability distribution, the time between successive arrivals must follow an exponential distribution.
6. The statistical distribution that is used for testing the difference between two population variances is the chi-square distribution.
7. Increasing the sample size causes the sampling distribution of the sample mean to have more dispersion.
8. A Type II error is committed when a true alternative hypothesis is mistakenly rejected.
9. If the coefficient of determination is a positive value, then the regression equation could have either a positive or a negative slope.
10. For the analysis of variance, "populations under consideration have equal means" is a required assumption.

**B. Multiple Choice (15 x 3% = 45%)**

**Exhibit 1: (Questions 1-2)**

Consider the following hypothesis test. $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_1: \sigma_1^2 \neq \sigma_2^2$ . If $n_1 = 31$, $s_1 = 0.2$, $n_2 = 8$, $s_2 = 0.1$, where $n_i$, $\sigma_i^2$, and $s_i$ are the sample size, population variance, and sample standard deviation from the $i$-th population (for $i$=1, 2).

1. Refer to **Exhibit 1**. Compute the value of the test statistic (F).
   (A) $F = 1$　(B) $F = 2$　(C) $F = 3$　(D) $F = 4$　(E) None of the above
2. Refer to **Exhibit 1**. What is the p-value? With the level of significance α=0.05, what is your decision for the hypothesis testing?
   (A) $0.01 \leq$ p-value $\leq 0.025$, reject $H_0$　(B) $0.025 \leq$ p-value $\leq 0.05$, reject $H_0$
   (C) $0.05 \leq$ p-value $\leq 0.1$, do not reject $H_0$　(D) $0.1 \leq$ p-value $\leq 0.2$, do not reject $H_0$
   (E) $0.2 \leq$ p-value $\leq 0.4$, do not reject $H_0$
3. In a simple linear regression analysis, if the total sum of squares (SST) = 4500 and the sum of squares errors (SSE) = 1575, then the correlation coefficient between the independent and dependent variables could be
   (A) 0.35 or -0.35　(B) 0.592 or -0.592　(C) 0.65 or -0.65　(D) 0.806 or -0.806　(E) 1

**Exhibit 2: (Questions 4-5)**

A ketchup manufacturer aims to determine whether the percentage of supermarket shoppers who believe the supermarket ketchup was as good as the national brand ketchup differed from 64%. Consider the hypothesis test: $H_0$: $p = 0.64$ versus $H_a$: $p \neq 0.64$.

4. Refer to **Exhibit 2**. [0.53, 0.68], [0.52, 0.67], [0.46, 0.61], [0.50, 0.65] are four 99% confidence interval estimates of $p$, based on 4 different samples, each of size 300. At the level of significance $\alpha =$

0.01, how many of them conclude failing to reject the null hypothesis?
(A) 4 (B) 3 (C) 2 (D) 1 (E) 0

**5.** Refer to **Exhibit 2.** A sample of 100 shoppers showed 52 stating that the supermarket brand was as good as the national brand. What is the value of the test statistic? Using the level of significance $\alpha = 0.01$, what is your conclusion?
(A) $-2.50$ ; fail to reject $H_0$ (B) $-2.50$ ; reject $H_0$ (C) $-2.402$ ; fail to reject $H_0$
(D) $-2.402$ ; reject $H_0$ (E) neither $-2.50$ nor $-2.402$ ; reject $H_0$

**Exhibit 3: (Questions 6-7)**
A consumer research group is interested in testing an automobile manufacturer's claim that a new economy model will travel at least 25 miles per gallon of gasoline. With a 5% level of significance and a sample of 36 cars. Assume that population variance is 9 miles per gallon.

**6.** Refer to **Exhibit 3.** What is the acceptance region based on the sample mean?
(A) $\overline{X} > 25$ (B) $\overline{X} > 24.1775$ (C) $\overline{X} > 22.5325$ (D) $\overline{X} \leq 22.5325$ (E) $\overline{X} \leq 24.1775$

**7.** Refer to **Exhibit 3.** What is the probability of committing a Type II error and power if the actual mileage is 23.5 miles per gallon?
(A) 0.95, 0.05 (B) 0.326, 0.674 (C) 0.0877, 0.9123 (D) 0.05, 0.95 (E) 0, 1

**Exhibit 4: (Questions 8-9)**
A survey found that the mean daily discretionary spending by Americans earning over \$90,000 per year was \$136 per day. The discretionary spending excluded home purchases, vehicle purchases, and regular monthly bills. Let $X$ be the discretionary spending per day and assume that a uniform probability density function applies with

$$f(x) = 0.00625, \text{ for } a \leq x \leq b.$$

**8.** Refer to **Exhibit 4.** Find the values of $a$ and $b$ for the probability density function.
(A) (56, 126) (B) (56, 216) (C) (126, 272) (D) (160, 320) (E) (272, 432)

**9.** Refer to **Exhibit 4.** What is the probability ($p$) that consumers in this group have daily discretionary spending between \$100 and \$200? Calculate the standard deviation ($\sigma$) of the distribution. What are the values of ($p$, $\sigma$)?
(A) (0.731, 2133.33) (B) (0.731, 46.188) (C) (0.542, 46.188)
(D) (0.625, 2133.33) (E) (0.625, 46.188)

**10.** The National Collegiate Athletic Association (NCAA) estimates that the yearly value of a full athletic scholarship at instate public universities is \$19,000. Assume the scholarship value is normally distributed with a standard deviation of \$2,100. For the 3% of athletic scholarships that are most valuable, how much are they worth?
(A) 14443 (B) 15052 (C) 17428 (D) 22948 (E) 23557

**Exhibit 5: (Questions 11-12)**

A simple linear regression model relating $X_i$ to $Y_i$, for $i = 1, \ldots, n$, provided the following information: $n = 14$, $\sum_{i=1}^{14} X_i = 630$, $\sum_{i=1}^{14} Y_i = 520.2$, $\sum_{i=1}^{14} X_i Y_i = 20940$, $\sum_{i=1}^{14} X_i^2 = 30300$, $\sum_{i=1}^{14} Y_i^2 = 22482.1$.

**11.** Refer to **Exhibit 5.** Use the least squares method to develop the estimated equation. Which one is correct?

(A) $\widehat{Y}_i = 94.134 - 1.266X_i$ (B) $\widehat{Y}_i = 94.134 + 1.266X_i$ (C) $\widehat{Y}_i = -19.822 + 1.266X_i$

(D) $\widehat{Y}_i = -19.822 - 1.266X_i$ (E) $\widehat{Y}_i = -407.487 + 9.881X_i$

**12.** Refer to **Exhibit 5.** Compute the mean square errors (MSE), mean square regression (MSR), and total sum of squares (SST). Which one is correct?

(A) MSE = 683.954, MSR = 2469, SST = 3152.954
(B) MSE = 1497.943, MSR = 3126.134, SST = 21101.447
(C) MSE = 26.820, MSR = 3126.134, SST = 3152.954
(D) MSE = 1552.704, MSR = 2469, SST = 21101.447
(E) MSE = 2.235, MSR = 3126.134, SST = 3152.954

**13.** Suppose that a study designed to collect new data on smokers and nonsmokers uses a preliminary estimate of the proportion who smoke of 0.20. With 99% confidence, how large a sample should be taken to estimate the proportion of smokers in the population with a margin of error of 0.02?

(A) 2171 (B) 2172 (C) 2653 (D) 2654 (E) 4147

**Exhibit 6. (Questions 14-15)**

The number of students per class (class size) data are collected from 50 college classes and summarized by the relative frequency as follows.

| Number of Students | 10-19 | 20-29 | 30-39 | 40-49 | 50-59 |
|---|---|---|---|---|---|
| Relative Frequency | 0.20 | 0.28 | ? | 0.14 | 0.04 |

**14.** Refer to **Exhibit 6.** The total sample size is 50. What is the cumulative frequency of class 30-39?

(A) 10 (B) 24 (C) 34 (D) 41 (E) 48

**15.** Refer to **Exhibit 6.** What is the sample mean number of students from the 50 college classes?

(A) 14.5 (B) 24.5 (C) 29.9 (D) 35.0 (E) 40.3

**C. Problems (35%)**

**1.** Consider a simple linear regression model:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \; i = 1, \ldots, n,$$

where $y_i$ is the response variable, $x_i$ is the independent variable, and $\varepsilon_i$ denotes the random error with $E(\varepsilon_i) = 0$ and $Var(\varepsilon_i) = \sigma^2$. Besides, $\varepsilon_i$'s for all individuals are assumed to be uncorrelated.

**(a)** Find the least squares estimators of regression coefficients, say the intercept $\beta_0$ and the slope $\beta_1$. **(8%)**

**(b)** Show that $E(MSE) = \sigma^2$, where 'MSE' is the mean square error. **(10%)**

**2.** Consider the following hypothesis test: $H_0$: $\mu \leq 20$ versus $H_a$: $\mu > 20$. The population standard deviation is 10. The sample size is 64. The power is 0.2 when the actual population mean is 21.4. What is the significance level $\alpha$ used in the test? **(8%)**

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**3.** **(9%)** An experiment of randomized block design has been conducted for four treatments with eight blocks. Complete the following analysis of variance (ANOVA) table. Please fill in **(a)-(i)**.

| Source of Variation | Sum of Squares | Degrees of Freedom | Mean Square | *F* |
|---|---|---|---|---|
| Treatments | 900 | **(b)** | **(f)** | **(i)** |
| Blocks | 400 | **(c)** | **(g)** | |
| Error | **(a)** | **(d)** | **(h)** | |
| Total | 1800 | **(e)** | | |

附表：Cumulative Probabilities for the Standard Normal Distribution

| *z* | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |

參考值：(Note that the probability indicates the area under the curve of the F distribution in the upper tail.)

| | | | |
|---|---|---|---|
| $F_{0.1}(30,7) = 2.56$ | $F_{0.05}(30,7) = 3.38$ | $F_{0.025}(30,7) = 4.36$ | $F_{0.01}(30,7) = 5.99$ |
| $F_{0.1}(7,30) = 1.93$ | $F_{0.05}(7,30) = 2.33$ | $F_{0.025}(7,30) = 2.75$ | $F_{0.01}(7,30) = 3.30$ |
| $F_{0.1}(31,8) = 2.38$ | $F_{0.05}(31,8) = 3.07$ | $F_{0.025}(31,8) = 3.89$ | $F_{0.01}(31,8) = 5.19$ |
| $F_{0.1}(8,31) = 1.88$ | $F_{0.05}(8,31) = 2.25$ | $F_{0.025}(8,31) = 2.64$ | $F_{0.01}(8,31) = 3.15$ |