

注意：考試開始鈴響前，不得翻閱試題，  
並不得書寫、畫記、作答。


國立清華大學 113 學年度碩士班考試入學試題

系所班組別：工程與系統科學系  
丙組

科目代碼：3101

考試科目：工程數學

### —作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 考試開始後，請於作答前先翻閱整份試題，是否有污損或試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「國立清華大學試場規則及違規處理辦法」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 113 學年度碩士班考試入學試題

系所班組別：工程與系統科學系碩士班 丙組(0531)

考試科目 (代碼)：工程數學 (3101)

共 2 頁，第 1 頁 \*請在【答案卷】作答

1. Find the solution of the given differential equation. Your answer should include the general solution  $y(x)$  and the exact solution when an initial or boundary condition is provided.

(a)  $x \frac{dy}{dx} - 2y = x^4 e^x$ . (5 %)

(b)  $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = -9x^2$ ,  $y(1) = 7$  and  $y(2) = 2 \ln 2$  (6 %)

(c)  $\frac{dy}{dx} = 2x + y - 2$ ,  $y(0) = 2$ . (6 %)

2. Find the series solution of the following differential equation about  $x = 0$ .

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy = 0. \quad (10\%)$$

You have to express the solution in the form of  $y(x) = C_1 y_1(x) + C_2 y_2(x)$ . To save time, you can only show the first four terms of  $y_1(x)$  and  $y_2(x)$ .

3. Use the Laplace transform to solve the problem and obtain  $y(t)$ .

$$\frac{dy}{dt} + \int_0^t y(\tau) d\tau = u(t-1) - u(t-2), \quad (8\%)$$

where  $y(0) = 0$  and  $u(t)$  is the unit step function.

4. Consider the matrix

$$M = \begin{bmatrix} 5 & 6 & -3 \\ -2 & -3 & 2 \\ 2 & 2 & 0 \end{bmatrix}.$$

(a) Find the determinant of  $M$  and obtain the inverse matrix  $M^{-1}$  (7 %).

(b) Estimate the eigenvalues and eigenvectors of  $M$  (8 %).

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考試科目 (代碼)：工程數學 (3101)

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5. Solve the wave equation  $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$  subject to the conditions:

$$u(0, t) = 0, \quad u(\pi, t) = 0, \quad t > 0$$

$$u(x, 0) = \frac{\pi}{2}x, \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} = 0, \quad 0 < x < \pi$$

(10%)

6. Use Fourier cosine transform to solve the steady-state temperature  $u(x, y)$  in a

plate from the equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ ,  $x \geq 0, y \geq 0$  subject to the conditions:

$$u_y(x, 0) = 0, x \geq 0 \text{ and } u(0, y) = \begin{cases} 30, & 0 \leq y \leq \pi \\ 0, & y > \pi \end{cases} \quad (10\%)$$

7. Solve temperature  $u(r, t)$  in a circular plate of radius  $c$  which is determined from the heat equation (10%)

$$\frac{\partial u}{\partial t} = k \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$

$$u(c, t) = 0, \quad t > 0$$

$$u(r, 0) = f(r), \quad 0 < r < c$$

[Hint: A general solution of the Bessel equation  $x^2 y'' + xy' + (\alpha^2 x^2 - \nu^2)y = 0$  is  $y = AJ_\nu(\alpha x) + BY_\nu(\alpha x)$ .]

8. (a) Use Stokes' theorem to determine the line integral of the vector function  $\mathbf{F} = (2x + y)\mathbf{i} + (x + y + z)\mathbf{j} + (y + 2z)\mathbf{k}$  along a closed curve  $C: (2 \cos \theta + \sin \theta)\mathbf{i} + (\cos \theta - 2 \sin \theta)\mathbf{j} + 3 \sin \theta \mathbf{k}$ ,  $0 \leq \theta < 2\pi$  (5%)

(b) Find the Laurent series of the complex function  $f(z) = \frac{1}{(z-i)^2(z+2i)}$ ,  $|z - i| > 3$  (5%)

(c) Determine the integral  $\oint_C \frac{\exp(z)}{z^3+z^2} dz$  where  $C$  is the circle of  $|z| = 2$ , oriented counterclockwise. (5%)

(d) Evaluate  $\int_0^{2\pi} \frac{1}{(2+\cos \theta)^2} d\theta$  (5%)