Macroeconomics exam: there are three short answer questions. Please write down your equations, solution process, and answers in the answering sheets. 請將解題過程及答案寫在答案紙上。

Question 1 (20 points)

Consider the representative agent model where each family is composed of an equal number of low-productivity members with permanent hourly productivity $w_L = e^{-a}$ and high-productivity members with productivity $w_H = e^a$ where $a > 0$ is a parameter. Suppose the labor income wh is subject to a progressive income taxation where the after-tax income is $\theta_0(wh)^{1-\theta_1}$. The parameter θ_0 captures the level of taxes and the parameter θ_1 captures the progressivity. In the economy, the family planner maximizes

$$\max_{c_t, h_{L,t}, h_{H,t}} \sum_{t=0}^{T} \beta^t \left[\left(\log(c_t) - \chi \frac{h_{L,t}^{1+\eta}}{1+\eta} \right) + \left(\log(c_t) - \chi \frac{h_{H,t}^{1+\eta}}{1+\eta} \right) \right]$$

subject to the family budget constraint

$$c_t + k_{t+1} = \theta_0 \left(e^{-a} h_{L,t}\right)^{1-\theta_1} + \theta_0 \left(e^{a} h_{H,t}\right)^{1-\theta_1} + k_t(1+r) + T$$

a) (5 points) Derive the first order conditions with respect to c_t

b) (5 points) Derive the first order conditions with respect to $h_{H,t}$ and $h_{L,t}$

c) (5 points) Derive $\frac{h_{H,t}}{h_{L,t}}$

d) (5 points) How the ratio of hours work $\frac{h_{H,t}}{h_{L,t}}$ change with progressivity θ_1?

Question 2 (30 points)

Consider the dynamic optimization problem:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \ln(c_t)$$

subject to

$$c_t + k_{t+1} = R_t k_t \ \forall t = 0, .., T$$
$$k_0 > 0 \ \text{given}$$

(a) (5 points) Derive the Euler equation

(b) (5 points) Assume that we have $\{R_t\}_{t=0}^{T}$ with R_t different at each t. Show that the ***consolidated*** budget constraint is

$$c_0 + \frac{1}{R_1} c_1 + \frac{1}{R_1 R_2} c_2 + \frac{1}{R_1 R_2 R_3} c_3 + \ldots + \frac{1}{R_1 \ldots R_T} c_T = k_0 R_0$$

(c) (5 points) Solve for c_0, c_1

(d) (5 points) Derive the general solution form of c_t

(e) (5 points) Suppose R_t increases, What happen to the consumption stream $\{c_0, c_1, \ldots, c_{t-1}\}$? (increase, decrease or unchange)

(f) (5 points) Find the intertemporal elasticity of substitution (i.e $\frac{d \ln(c_{t+1}/c_t)}{d \ln R_{t+1}}$)

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Question 3 (50 points)

Heterogeneity in an endowment economy

Consider an endowment economy with two periods, t and $t+1$. Suppose there are two types of households. Among the population, 60% are type 1 agents and 40% are type 2 agents. Normalize total population to 1. Households differ in their endowments, but they have the same preference.

Endowments:

Type 1 agents: $Y_t^1 = 7, Y_{t+1}^1 = 4$

Type 2 agents: $Y_t^2 = 3, Y_{t+1}^2 = 10$.

Each agent has the same utility function:

$$\max_{c_t, c_{t+1}} \ln c_t + \beta \ln c_{t+1},$$

subject to

$$c_t^i + \frac{c_{t+1}^i}{1+r} = Y_t^i + \frac{Y_{t+1}^i}{1+r_t},$$

where $\beta = 1$ and i =1 or 2, indicating each type of agents. In the following questions, round off your answer to two decimal places.

(a) (6 points) What are aggregate endowments Y_t and Y_{t+1}?

(b) (10 points) Derive the optimal consumption function at time t for each type of agents.

(c) (4 points) Using the optimal consumption in (b), what is the aggregate consumption at time t?

(d) (6 points) What is the market clearing condition in this endowment economy?

(e) (8 points) What is the value of the equilibrium interest rate r_t?

(f) (8 points) In the general equilibrium, derive the equation of the IS curve, which is the relationship (an equation) between the equilibrium interest rate r_t and the aggregate endowments Y_t and Y_{t+1} (write down r_t as a function of Y_t and Y_{t+1}).

(g) (8 points) Now, suppose every agent has the identical endowment that is equal to the aggregate endowment in (a) at time t and $t+1$, what is the value of the equilibrium interest rate r_t?