國立交通大學 101 學年度碩士班考試入學試題

科目：數學(5081)(5041)(5051)　　考試日期:101 年 2 月 16 日　第 1 節

系所班別：交通運輸研究所、運輸科技與管理學系甲、乙組-般生　　第 1 頁, 共 1 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1.Evaluate the following problems:

(1) $\int \frac{\cos y dy}{\sin^2 y + \sin y - 6}$ (10 points)

(2) $\int (x^2 - 5x)e^x dx$ (10 points)

(3) $\lim_{x\to\infty} 3(x - \sqrt{x^2 + x})$ (10 points)

2. Given that $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} \cdot W \cdot \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, solve for the matrix W. (10 points)

3. Under what conditions on x will the matrix $\begin{bmatrix} x & \sqrt{2} & 0 \\ \sqrt{2} & x & \sqrt{2} \\ 0 & \sqrt{2} & x \end{bmatrix}$ fail to have an inverse? (10 points)

4. Determine whether or not the vectors $\begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \end{bmatrix}$ are linearly dependent. (10 points)

5. Find volumes of the solids of revolution if the region bounded by the curves $y = 2x - x^2$, $y = x^2 - 2x$ is rotated around:
(1) the y-axis (10 points)
(2) $y = x + 2$ (10 points)

6. What is the maximum of the function $f(x) = 8x^3 - 12x^2 + 6x - 1$ in [-1, 2]? (10 points)

7. Using the *squeeze theorem* to prove that $\lim_{x\to 0} x \sin x = 0$ (10 points)