考試科目	統計學	系所別	風險管理與保險學系 精算科學組	考試時間	2 月 6 日(二) 第 4 節

1. (20 pts) Please explain the following items.

   (a) (10 pts) Bayesian Estimates and Minimax Principle

   (b) (10 pts) Uniformly Most Powerful Tests

2. (10pts) An application processor for a small life insurance and annuity company processes all applications for both products. Applications arrive according to a Poison process at a rate of 6 per hour and any one application has probability 0.4 of being for an annuity policy.

   Calculate the probability that the next 3 applications received are for life insurance. ______ (A) 0.18 (B) 0.22 (C) 0.26 (D) 0.30 (E) 0.34

3. (10 pts) Let $X_1, X_2, \cdots, X_{30}$ be a random sample from a uniform distribution on $(0,1)$. Let $X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq \cdots \leq X_{(30)}$ be the order statistics. Determine the cumulative distribution Function $F_Y(y)$, of $Y = X_{(4)}$ for $0 < y < 1$.

4. (10 pts) Let $\{x_i\}_{i=1}^n$ be an independent and identically distributed sample from a Poisson distribution with parameter λ. Calculate the maximum likelihood estimator of the parameter $E(X_i^2)$, denoted as T. Explain the asymptotic distribution of T (Please specify the distribution name and parameters).

5. (15 pts) Let the conditional probability that a person insured at age x will attain age $x+t$ is given by

$$\left(\frac{1+x}{1+x+t}\right)^3, \forall t > 0$$

   (a) (5 pts) Calculate the probability that this insured person will die within two years at age 20.

   (b) (10 pts) Estimate this insured person's expected future life time at age 41.

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6. (15pts) Let X have the p.d.f.

$$f(x;\theta) = \begin{cases} \theta^x(1-\theta)^{1-x}, & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}.$$

We test the simple hypothesis $H_0 : \theta = \frac{1}{4}$ against the alternative composite hypothesis $H_1 : \theta < \frac{1}{4}$ by taking a random sample of size 10 and rejecting H_0 if and only if the observed values $x_1, x_2, \cdots, x_{10}$ of the sample items are such that $\sum_{i=1}^{15} x_i \leq 1$. Find the power function $K(\theta)$, $\theta \leq \frac{1}{4}$, of this test.

7. (20pts) A drunken man wanders around randomly in a large space. At each step, he moves one unit of distance North, South, East, or West, with equal probabilities. Choose coordinates such that his initial position is $(0,0)$ and if he is at (x,y) at some time, then one step later he is at $(x,y+1)$, $(x,y-1)$, $(x+1,y)$, or $(x-1,y)$. Let (X_n, Y_n) and R_n be his position and distance from the origin after n steps, respectively.

General hint: note that X_n is a sum of random variables with possible values $-1, 0, 1$, and likewise for Y_n, but be careful throughout the problem about independence.

(a) (5 pts) Determine whether or not X_n is independent of Y_n (explain clearly).

(b) (10 pts) Find $Cov(X_n, Y_n)$. (Please simplify.)

(c) (5 pts) Find $E(R_n)$. (Please simplify.)

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。