

考試科目	微積分	系所別	國際管理碩士班 金融系(10)	考試時間	2月6日(二)第二節
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Problem 1 (20 points) (10 % each)

Evaluate the integral

(a)

$$\int x^\alpha \ln x \, dx, \alpha \in \mathbb{R}.$$

(b)

$$\int_1^\infty \frac{x^2 - 3}{(x^2 - 2x + 3)(x^2 + 2x + 3)} \, dx.$$

Problem 2 (10 points)

(a) Evaluate the limit

$$I_m = \lim_{n \rightarrow \infty} \sum_{i=1}^{nm} \frac{i^2 n^3}{n^6 + i^6},$$

where m is a positive integer.

(b) Compute $\lim_{m \rightarrow \infty} I_m$.

Problem 3 (10 points)

Suppose that the function $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) , and $0 < a < b$. If $f(a) = ka$, $f(b) = kb$ for some k , show that there exists $c \in (a, b)$ such that the tangent line of $y = f(x)$ at c passes through the origin.

Problem 4 (10 points)

If $y = f(u)$ and $u = g(x)$, where f and g are twice differentiable functions, with $g(0) = 1$, $f(1) = 2$, $g'(0) = 2$, $f'(1) = -1$, $g''(0) = 1$, and $f''(1) = 3$, Find $\frac{d^2 y}{dx^2} \Big|_{x=0}$.

備註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。

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Problem 5 (10 points)

Evaluate the the iterated integral

$$\int_0^a \int_x^a \sin(y^2) dy dx, a > 0.$$

Problem 6 (10 points)

Find the values of a for which the improper integral

$$\int_1^{\infty} \frac{dx}{x^a(1+\sqrt{x})}$$

converges.

Problem 7 (15 points)

Let $I_n = \int_0^{\infty} x^n e^{-x} dx.$

- (a) Find the recursive relation between I_n and $I_{n-1}.$
- (b) Compute $I_3.$
- (c) Find the general formula of $I_n.$

Problem 8 (15 points)

Find the local maximum and minimum values and saddle point(s) of the function

$$f(x, y) = x^3 - y^3 + 3x^2 + 3y^2 - 9x.$$

備註

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- 二、試題請隨卷繳交。