國立政治大學 學年度 碩士班暨碩士在職專班 招生考試試題 113

第 | 頁,共2頁

試科目 经公主差分

信任のG(できょうは時間 ア月6日(ニ)第二節

Problem 1 (20 points) (10 % each)

Evaluate the integral

(a)

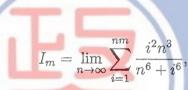
$$\int x^{\alpha} \ln x \, dx, \, \alpha \in R.$$

(b)

$$\int_{1}^{\infty} \frac{x^2 - 3}{(x^2 - 2x + 3)(x^2 + 2x + 3)} dx$$

Problem 2 (10 points)

(a) Evaluate the limit



where m is a positive integer.

(b) Compute $\lim_{m\to\infty} I_m$.

Problem 3 (10 points)

Suppose that the function f(x) is continuous on [a, b] and differentiable on (a, b), and 0 < a < b. If f(a) = ka, f(b) = kb for some k, show that there exists $c \in (a, b)$ such that the tangent line of y = f(x) at c passes through the origin.

Problem 4 (10 points)

If y = f(u) and u = g(x), where f and g are twice differentiable functions, with g(0) = 1, f(1) = 2, g'(0) = 2, f'(1) = -1, g''(0) = 1, and f''(1) = 3, Find $\frac{d^2y}{dx^2}|_{x=0}$.

考試科目

级稳

系所别 3 4篇210 考試時間 2 月 6 日(二)第二節

Problem 5 (10 points)

Evaluate the the iterated integral

$$\int_0^a \int_x^a \sin(y^2) dy dx, \ a > 0.$$

Problem 6 (10 points)

Find the values of a for which the improper integral

$$\int_{1}^{\infty} \frac{dx}{x^a (1 + \sqrt{x})}$$

converges.

Problem 7 (15 points)

Let
$$I_n = \int_0^\infty x^n e^{-x} dx$$
.

- (a) Find the recursive relation between I_n and I_{n-1}
- (b) Compute I_3 .
- (c) Find the general formula of I_n .

Problem 8 (15 points)

Find the local maximum and minimum values and saddle point(s) of the function

$$f(x,y) = x^3 - y^3 + 3x^2 + 3y^2 - 9x.$$

二、試題請隨卷繳交。