

考試科目	數理統計學	系所別	統計學系	考試時間	2月6日(二)第二節
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Note:  $P(Z \leq 2) = .977$ ,  $P(Z \leq 1.96) = .975$ ,  $P(Z \leq 1.645) = .95$ ,  $\log(.05) = -2.99$ ,  $\log(.025) = -3.68$

- (10 points) Let  $X$  be a random variable with  $P(X \geq x) = 1/2^x$ , for  $x = 0, 1, 2, \dots$ . Find the expected value of  $X$ .
- (15 points) Let  $X_1, \dots, X_{36}$  be a random sample from an exponential distribution  $\text{EXP}(1)$ . Let  $S = \sum_{i=1}^{36} X_i$ . Give an approximation for  $P(S > 48)$ .
- (30 points) Suppose that  $X_1, \dots, X_{10}$  are independent random variables and  $X_k$  follows an exponential distribution  $\text{EXP}(k)$ , for  $k = 1, \dots, 10$ . The pdf of  $X_k$  is

$$f(x) = ke^{-kx}, \quad x > 0.$$

Let  $(X_{(1)}, X_{(2)}, \dots, X_{(10)})$  denote the order statistics, with  $X_{(1)} < X_{(2)} < \dots < X_{(10)}$ .

- (15 points) Find the probability distribution of  $X_{(1)}$ .
  - (15 points) Find the probability  $P(X_{(1)} = X_{(10)})$ .
- (45 points) Let  $X_1, X_2, \dots, X_n$  be an i.i.d. sample from a probability distribution with pdf  $f(x; \theta) = \exp[-(x - \theta)]$ , for  $x > \theta$ .
    - (15 points) Find a sufficient statistic for  $\theta$ .
    - (15 points) Find a UMP (uniformly most powerful) test of size  $\alpha$  for  $H_0: \theta \leq \theta_0$  versus  $H_1: \theta > \theta_0$ .
    - (15 points) (continued from (b)) Suppose that the observed data are

$$(x_1, \dots, x_{10}) = (5.81, 6.28, 8.55, 5.75, 5.80, 6.83, 5.10, 5.42, 5.26, 5.93).$$

At  $\alpha = 0.05$ , test  $H_0: \theta \leq 5$  versus  $H_1: \theta > 5$ .

備註

- 作答於試題上者，不予計分。
- 試題請隨卷繳交。