國立政治大學 113 學年度 碩士班暨碩士在職專班 招生考試試題

考 試 科 目	線性代數	系 所 別	應用數學系	考 試 時 間	2 月 5 日(一) 第 四 節

Show all your work to earn the credits.

1. (10 points) Let V be a vector space over a field F and let $v_1, v_2, v_3 \in V$ such that $v_1 \neq 0$, $v_2 \notin span(\{v_1\})$ and $v_3 \notin span(\{v_1, v_2\})$. Show that v_1, v_2 and v_3 are linearly independent.

2. Given the linear system:

$$(k+2)x + 2ky - z = 1$$
$$-x + 2y - kz = k$$
$$y + z = k$$

   (a) (10 points) Find the value of k such that the system has infinitely many solutions.

   (b) (10 points) Find the form of the solutions for the system with the value of k found in (a).

3. Let P_3 be the vector space over the real field $\mathbb{R}$ which contains all polynomials of degree less than or equal to 3 with real coefficients. Let

$$W = \left\{p(x) \in P_3 : p'(-1) = 0 \text{ and } p''(1) = 0\right\}$$

   where $p'(x)$ and $p''(x)$ denote the first- and second-order derivatives of $p(x)$, respectively.

   (a) (10 points) Show that W is a subspace of P_3.

   (b) (10 points) Find a basis for W.

4. A matrix M is said to be skew-symmetric if

$$M^T = -M,$$

   where M^T is the transpose of M.

   (a) (10 points) Show that the determinant of an $n \times n$ skew-symmetric matrix is zero if n is odd.

   (b) (10 points) Show that each eigenvalue of the real skew-symmetric matrix M is either 0 or a purely imaginary number.

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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5. Let

$$A = \begin{bmatrix} 3 & 3 & 2 \\ 0 & 1 & 0 \\ 2 & -2 & 3 \end{bmatrix}$$

(a) (10 points) Compute the eigenvalues and the eigenvectors of A over the field $\mathbb{R}$.

(b) (5 points) Prove or disprove that A is diagonalizable over $\mathbb{R}$.

(c) (5 points) Prove or disprove that A is diagonalizable over $\mathbb{Z}_5$, where $\mathbb{Z}_5$ is the finite field of 5 elements in which the addition and multiplication are the same in integers except taking modulo 5.

6. (10 points) Let A be a matrix such that there are exactly two vectors

$$(1, 2, 1)^T \quad \text{and} \quad (1, 1, 0)^T$$

as the eigenvectors associated with the eigenvalue 7. Assume that the trace of A is 2. Find the determiniant of A.

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