

考試科目	微積分	系所別	應用數學系	考試時間	2月5日(一)第三節
1. (24 points) Evaluate the integral.					
A. (8 points) $\int_0^{\pi/2} \sin^3 \theta \cos^2 \theta d\theta$					
B. (8 points) $\int_0^1 \int_{x^2}^1 \sqrt{y} \sin y dy dx$					
C. (8 points) $\iiint_E \frac{1}{x^3} dV$ , where $E = \{(x, y, z) : 0 \leq y \leq 1, 0 \leq z \leq y^2, 1 \leq x \leq z + 1\}$					
2. (8 points) If $f(x) = \int_0^x x^2 \sin(t^2) dt$ , find $f'(x)$ .					
3. (8 points) Let $h(x) = (\sin x)^x$ . Show that $h'(x) = (\sin x)^x (x \cot x + \ln \sin x)$ .					
4. (10 points) Evaluate the following improper integral $\int_0^4 \frac{\ln x}{\sqrt{x}} dx$ or show it is divergent.					
5. (10 points) Let $f(x, y) = \begin{cases} \frac{x^3y - xy^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$					
Find the value of the derivative $f_{xy}(0, 0)$ .					
6. (10 points) If the function $f$ satisfies the equation $f(tx, ty) = t^n f(x, y)$ for all $t$ , where $n$ is a positive integer and $f$ has continuous partial derivatives. Show that					
$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = n f(x, y).$					
7. (10 points) Suppose that $\lim_{x \rightarrow a} f(x) = \infty$ and $\lim_{x \rightarrow a} g(x) = c$ , where $c > 0$ is a real constant. Prove that					
$\lim_{x \rightarrow a} [f(x)g(x)] = \infty.$					
8. (10 points) Show that, for $x > 0$ ,					
$\frac{x}{1+x^2} < \tan^{-1} x < x.$					
9. (10 points) A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}$ , $a_{n+1} = \sqrt{2 + a_n}$ ( $n = 1, 2, \dots$ ). Show that the limit					
$\lim_{n \rightarrow \infty} a_n$ exists and find the value of this limit.					
備註	一. 作答於試題上者，不予計分。 二. 試題請隨卷繳交。				