國立成功大學

112學年度碩士班招生考試試題

編　號：204

系　所：電機資訊學院-資訊聯招

科　目：計算機數學

日　期：0206

節　次：第 3 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、離散數學 (50%)

1. (10 points) Among the 900 three-digit integers (from 100 to 999) those such as 131, 222, 303, 717, 848, and 969, where the integer is the same whether it is read from left to right or from right to left, are called palindromes. Without actually determining all of these three-digit palindromes, we would like to determine their sum. Please calculate the sum of these palindrome ranging from 100 to 999.

2. (10 points) Determine the generating function for the sequence 1, 1, 1, …, 1, 0, 0, 0,…, where the first n+1 terms are 1.

3. (10 points) How many bijective functions are there from a finite set A to a finite set B where $|A| = |B| = n$?

4. (10 points) Solve the recurrence relation
$a_{n+2} - 5a_{n+1} + 6a_n = 2, n \geq 0, a_0 = 3, a_1 = 7.$

5. (10 points) Let $a, b \in \mathbf{Z}$ and let $2a + 3b$ be a multiple of 17. (For example, we could have $a = 7, b = 1$; and $a = 4, b = 3$ also works.) Determine that the following statement is true or false: 17 divides $9a + 5b$.

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二、線性代數 (50%)

6. (9%) Determine the coordinate vector of $\boldsymbol{w} = (8, -4, -12)$ with respect to the basis {(1, 1, 1), (1, 5, -3), (2, 2, 1)}.

7. (9%) Let $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ be an orthonormal basis for an inner product space V. If $\boldsymbol{x} = c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + c_3\boldsymbol{u}_3$ is a vector with the properties $\|\boldsymbol{x}\| = 5$, $\langle \boldsymbol{u}_1, \boldsymbol{x} \rangle = 4$, and $\boldsymbol{x} \perp \boldsymbol{u}_2$, then what are the possible values of c_1, c_2, c_3?

8. Let A be a 5 by 7 matrix with rank 4.
   (a) (5%) What is the dimension of the solution space of $A\boldsymbol{x} = \boldsymbol{0}$?
   (b) (5%) Is $A\boldsymbol{x} = \boldsymbol{b}$ consistent for all vectors $\boldsymbol{b}$ in R^5? Explain.

9. (10%) Given that the characteristic polynomial of a matrix A is $p(\lambda) = (\lambda + 1)(\lambda - 2)^2(\lambda + 3)^2$, find $\det(A^{-1})$.

10. True or False
    (a) (3%) If A is an $n \times n$ matrix whose eigenvalues are all nonzero, then A is nonsingular.
    (b) (3%) If A is a 5×5 matrix of rank 1 and $\lambda = 0$ is an eigenvalue of multiplicity 4, then A is diagonalizable.
    (c) (3%) If A and B are row equivalent matrices, then their determinants are equal.
    (d) (3%) If A is an invertible $n \times n$ matrix, then $rank(A^T) = 0$