科目：離散數學與資料結構　　系所：資訊工程學系
考試時間：100 分鐘　　　　本科原始成績：100 分　　　是否使用計算機：否

1. (a) (5%) Find the number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 8$, $x_i \in Z$, $x_i \geq 0$, $1 \leq i \leq 5$.

(b) (5%) Find the number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 8$, $x_i \in Z$, $x_i > 0$, $1 \leq i \leq 5$.

2. When John goes to school, he goes through a sequence of six road traffic lights, L_1, L_2, L_3, L_4, L_5, and L_6, each has red, yellow, or green lights.

(a) (5%) In how many ways, does John have the six traffic lights on his way to school, for example, a sequence of red, red, yellow, green, yellow, and green lights?

(b) (5%) In how many ways, does John have the six traffic lights on his way to school with at least one red light, at least one yellow light, and at least one green light?

3. (a) (3%) What is the Pigeonhole Principle?

(b) (7%) For any $n \in Z^+$, show that there exist two distinct integers $x, y \in \{3^n, 4^n, 5^n, ..., 10^n\}$ such that $7 | (x - y)$.

4. (20%) Please propose a method, including a data structure and an algorithm, to show how to reduce the space to store the following data sequence.

<div align="center">ppxxxxxxpaaabapaaabff</div>

5. (10%) Please discuss what would happen if we apply the following assignment statement to the linked list shown below (that is, "**(*a).link**" is assigned to "**a**").

<div align="center">

a ← (*a).link
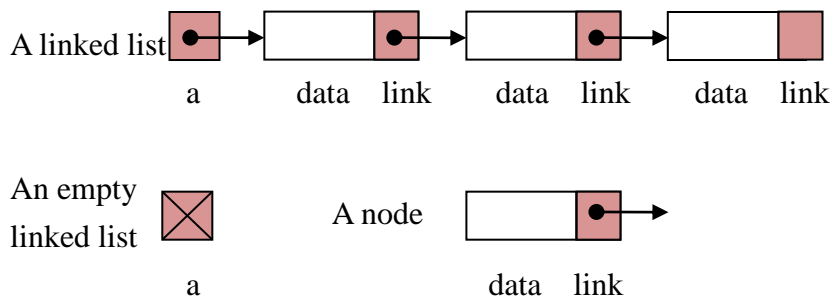
</div>

A linked list

An empty linked list　　　A node

6. (10%) Write a code segment that inserts into a circular doubly linked list the node to which B points before the node to which A points.

7. (30%) The following function has two maxima — one of which is a local maximum, and the other one is the global maximum (i.e., the pair of (x, y) that yields a local or global maximum of f):

$$f(x,y) = (1-x)^2 e^{-x^2-(y+1)^2} - (x - x^3 - y^3)\, e^{-x^2-y^2}$$

where
$$-3 \le x \le 3$$
$$-3 \le y \le 3$$

Please propose a data structure and corresponding algorithm to find the x and y that generate the global maximum of f. (Note: the goal is to examine how you solve the problem, rather than asking for the values of x and y that yield the maximum of f. Your proposed data structure is 15%, and the algorithm is another 15%.)