**國立臺灣科技大學 112 學年度碩士班招生試題**

系所組別：自動化及控制研究所碩士班
科　　目：工程數學

**（ 總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分 ）**

1. (13%) Solve the initial value problem $y'' - y = 5\sin^2(x); y(0) = 2, y'(0) = -4$.

2. (13%) Find the Laplace transform of $f(t) = t$ when $-1 < t < 1$, and $f(t + 2) = f(t)$.

3. (13%) Find the general solution of the differential equation $y' = \frac{y}{x+y}$.

4. (11%) Let $f(x) = x - x^2$ for $-\pi \le x \le \pi$. Find the Fourier series of $f(x)$ on $[-\pi, \pi]$.

**（ 總分為100分；所有試題務必於答案卷內頁依序作答，否則不予計分 ）**

5. (20%) Evaluate the Fourier transform of the continuous-time signal:

$$x(t) = t\frac{d}{dt}\{\, te^{-2t}\cos(t)\,u(t)\,\} \quad \text{where} \quad u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}.$$

6. (20%) Let A be the matrix $A = \begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   (1) (10%) Prove that A is an orthogonal matrix.
   (2) (10%) Find the eigenvalues of A.

7. (10%) Define operator $T(a_1, a_2) = (-2ia_1 + 3a_2,\ a_1 - a_2)$, which maps a two-dimensional complex vector space to another two-dimensional complex vector space $(T: \mathbb{C}^2 \to \mathbb{C}^2)$. Find $T^*(a_1, a_2)$, where $T^*$ is the adjoint of the operator T.