國立中山大學112學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：普通物理【材光系碩士班選考、材料前瞻應材碩士班選考、材光聯合碩士班選考】

－作答注意事項－

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答(不得另攜帶紙張，亦不得使用應考證空白處作為計算紙使用)。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶具有通訊、記憶或收發等功能或其他有礙試場安寧、考試公平之各類器材、物品（如鬧鈴、行動電話、電子字典等）入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學112學年度碩士暨碩士專班招生考試試題

科目名稱：普通物理【材光系碩士班選考、材料前瞻應材碩士班選考、材光聯合碩士班選考】
題號：488005
※ 本科目依簡章規定「不可以」使用計算機（問答申論題）

Problem 1. [Mechanics: 40 points]

In the following, we consider particles only move in x-direction.

(a) [5 points] There are two springs, one with the elastic constant k_1, and the other with elastic constant k_2. If the two springs are in series, what is the effective elastic constant k_{eff}?

(b) [5 points] Consider 2 identical atoms with mass m, connected by a chemical bond modeled as a spring with elastic constant k_σ (a sigma bond). The center of mass is stationary (not moving). Solve the angular frequency of the vibration motion of the 2 atoms. (hint: it might be easier if using the result of (a))

(c) [5 points] There are two springs, one with the elastic constant k_1, and the other with elastic constant k_2. If the two springs are in parallel, what is the effective elastic constant k_{eff}?

(d) [5 points] Consider 2 identical atoms with mass m, connected by 2 chemical bonds (that is, a double bond) modeled as 2 springs with elastic constant k_σ and k_π. The center of mass is stationary (not moving). Solve the angular frequency of the vibration motion of the 2 atoms. (hint: use the result of (c))

(e) [10 points] Consider 2 non-identical atoms with mass M and m (such as a carbon and an oxygen atom), connected by a chemical bond modeled as a spring with elastic constant k_σ (a sigma bond). The center of mass is stationary (not moving). Solve the angular frequency of the vibration motion of the 2 atoms.

(f) [5 points] Consider 2 non-identical atoms with mass M and m (such as a carbon and an oxygen atom), connected by 3 chemical bonds modeled as 3 springs with the same elastic constant k_σ (a sigma bond). (i) solve the angular frequency of the vibration motion of the 2 atoms, and (ii) compare the obtained frequency with the single bond case above.

(g) [5 points] Carbon dioxide CO_2 can be modeled as one carbon atom and two oxygen atoms linked by springs (double bonds). How many are the vibrational modes? Please give a proper argument, not just giving the answer.

Problem 2. [Electromagnetism: 35 points]

The electric field at a distance $\boldsymbol{\mathcal{r}}$ from a point charge Q is:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\mathcal{r}^2}\hat{\boldsymbol{\mathcal{r}}},$$

where $\hat{\boldsymbol{\mathcal{r}}}$ is the unit vector along the vector $\boldsymbol{\mathcal{r}}$. The electric potential is:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\mathcal{r}}$$

Consider the following questions in the vacuum, and **use the units in the above formula**.

(a) [5 points] If there are two point charges $-q$ and q, separated by a distance d, as shown in the figure below. The magnitude of the dipole moment p of these two point charges is then $p = qd$. Write down the electric potential at **r** (the origin is at the middle of the two charges), using those quantities given in the figure.

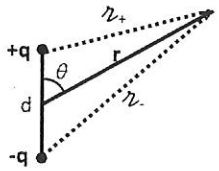

Figure 1

(b) [5 points] Prove $r_{\pm}^2 = r^2\left(1 \mp \frac{d}{r}\cos\theta + \frac{d}{4r^2}\right)$.

(c) [5 points] Use the approximation $(1+\delta)^\alpha \approx 1 + \alpha\delta$ if $\delta \ll 1$ to show: when $r \ll d$ (measuring the potential far away from the dipole), $\frac{1}{r_\pm} \approx \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right)$.

(d) [10 points] When $r \ll d$ (measuring the potential far away from the dipole), show the electric potential $V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$.

(e) [10 points] From the above result, calculate the electric field far away from the dipole using: $\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}\right)$.

Problem 3. [Thermodynamics: 10 points]

(a) [5 points] A gas is enclosed in a cylinder with a moveable piston, and follows the equation:

$$P^3V^5 = \text{constant},$$

where P is the pressure, and V is the volume of the gas. Find the work done when the volume is compressed from V_A to V_B, with a known initial pressure P_A (the final pressure P_B is not known).

(b) [5 points] The Maxwell-Boltzmann distribution of the speed v of the particle is:

$$f(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} 4\pi v^2 e^{\frac{-mv^2}{2k_B T}},$$

where m is the mass of the particle, k_B is the Boltzmann constant, and T is the temperature. Calculate the most probable speed v_p, which corresponds to the maximum value of $f(v)$.

Problem 4. [Waves/Optics/Modern physics: 15 points]

If a propagating wave along the x direction has the form $f(x,t) = Ae^{i(kx-\omega t+\phi)}$, where x is the position, t is the time, A is a constant amplitude and ϕ is a constant phase.

(a) [5 points] (i) Show that this wave function satisfies the wave equation

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

(ii) express the velocity v using A, k, ω, or ϕ.

(b) [5 points] The wave is a periodic function of the time, so $f(x,t_0) = f(x,t_0+T)$ for arbitrary t_0 (where T is the period). Use this relation, express the period T with A, k, ω, and ϕ.

(c) [5 points] The wave is a periodic function of the position, so $f(x_0,t) = f(x_0+\lambda,t)$ for arbitrary x_0 (where λ is the wave length). Use this relation, express the wave length λ with A, k, ω, and ϕ.