國立臺灣科技大學112學年度碩士班招生試題

系所組別：化學工程系碩士班

科　　目：工程數學與輸送現象

（總分為100分；所有試題務必於答案卷內頁依序作答，否則不予計分）

1. **(38%)** A naphthalene (MW=128 g/mol) sphere is kept at a uniform temperature of 27℃ and is suspended in still air at 1 atm (1×10^5Pa) by a fine wire. The initial radius r_I= 2.00 mm. The saturated vapor pressure of naphthalene at 27℃ is P_{AI}=0.6 mmHg, and the density of solid naphthalene is 1.14 g/cm^3. The diffusivity of naphthalene in the air is $7\text{x}10^{-6}$ m^2/s. The gas constant, R, is 8.314 $\frac{J}{mol\cdot K}$.

   (a) (10%) Derive the evaporation rate (kg/s) of naphthalene and explain the assumptions you made to simplify the governing equation.

   (b) (18%) Derive the profile of p_A.

   (c) (10%) What is the time for the completed evaporation of this naphthalene sphere?

2. **(12%)** Choose the CORRECT one(s) from the following descriptions.

   (a) In a backward-feed multiple-effect evaporator, the high temperatures in the early effects reduce the viscosity, resulting in good heat and mass transfer.

   (b) The percentage humidity is defined as the actual vapor pressure of water in the air divided by the water vapor pressure which is saturated at the same temperature.

   (c) The bound moisture in solids has lower vapor pressure than water at the same temperature.

   (d) In the design of distillation tower, the q line represents heat condition of the feed, and q is defined as q = (molar latent heat of feed vaporization)/(heat needed to vaporize 1 mol of feed at entering conditions)

   (e) The reflux of distillation increases the purity of the overhead product, and decreases the energy cost.

   (f) After the break-point time is reached in an adsorption bed, the solute concentration rises very rapidly, and the bed is then judged ineffective soon.

3. **(30%)**

   (5%) (a) In fluid mechanics, Newton's viscosity law is mainly used to describe the relationship between fluid velocity gradient and shear stress. Consider a one-dimensional flow, if the flow direction of the fluid is in the r direction, and the direction of momentum transport is in the z direction, please write the corresponding Newton's viscosity law.

   (5%) (b) The shear stress can be regarded as the flux of which physical quantity ?

   (5%) (c) Stream function (Ψ) is useful for flow visualization. It applies to incompressible two-dimensional flow situation. For example, consider a two-dimensional x-y flow in Cartesian rectangular coordinates, we can find the relationship between (v_x, v_y) and Ψ by using continuity equation as following:

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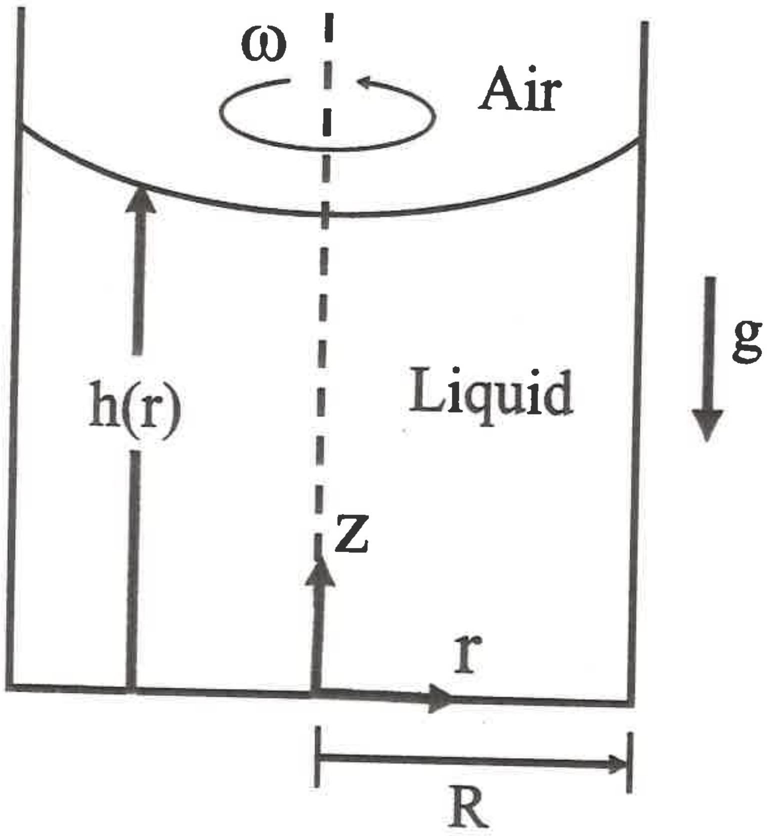
$\because \nabla \bullet \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$ and Let $v_x = \frac{\partial \Psi}{\partial y}$

substitute v_x into $\nabla \bullet \vec{v} \Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial \Psi}{\partial y}\right) + \frac{\partial v_y}{\partial y} = 0$

So, we can find $v_y = -\frac{\partial \Psi}{\partial x}$

Now, we consider the flow pattern in r-θ plane of spherical coordinates. Please find the relationship between (v_r, v_θ) and Ψ.

(15%) (d) The following figure shows a system with a gas-liquid interfaces, consisting of a liquid in an open container of radius R that is rotated at a constant angular velocity ω. Please determine the steady interface height, h(r), assuming that the ambient air pressure is P_0 and the initial volume of the liquid in the tank is V_0.



4. **(20%) Please answer the following questions.**

(5%) (a) Fourier law of heat conduction can be expressed as $\frac{q}{A} = -k\nabla T$. Please explain the physical meaning of negative symbol "–".

(15%) (b) Consider a two-dimensional heat transfer in x-y plane as shown in following figure. The temperatures of the right-hand side and left-hand side are T_a and T_b, respectively. Consider the thermal conductivity, the width and the thickness of the fin are k, t_f and

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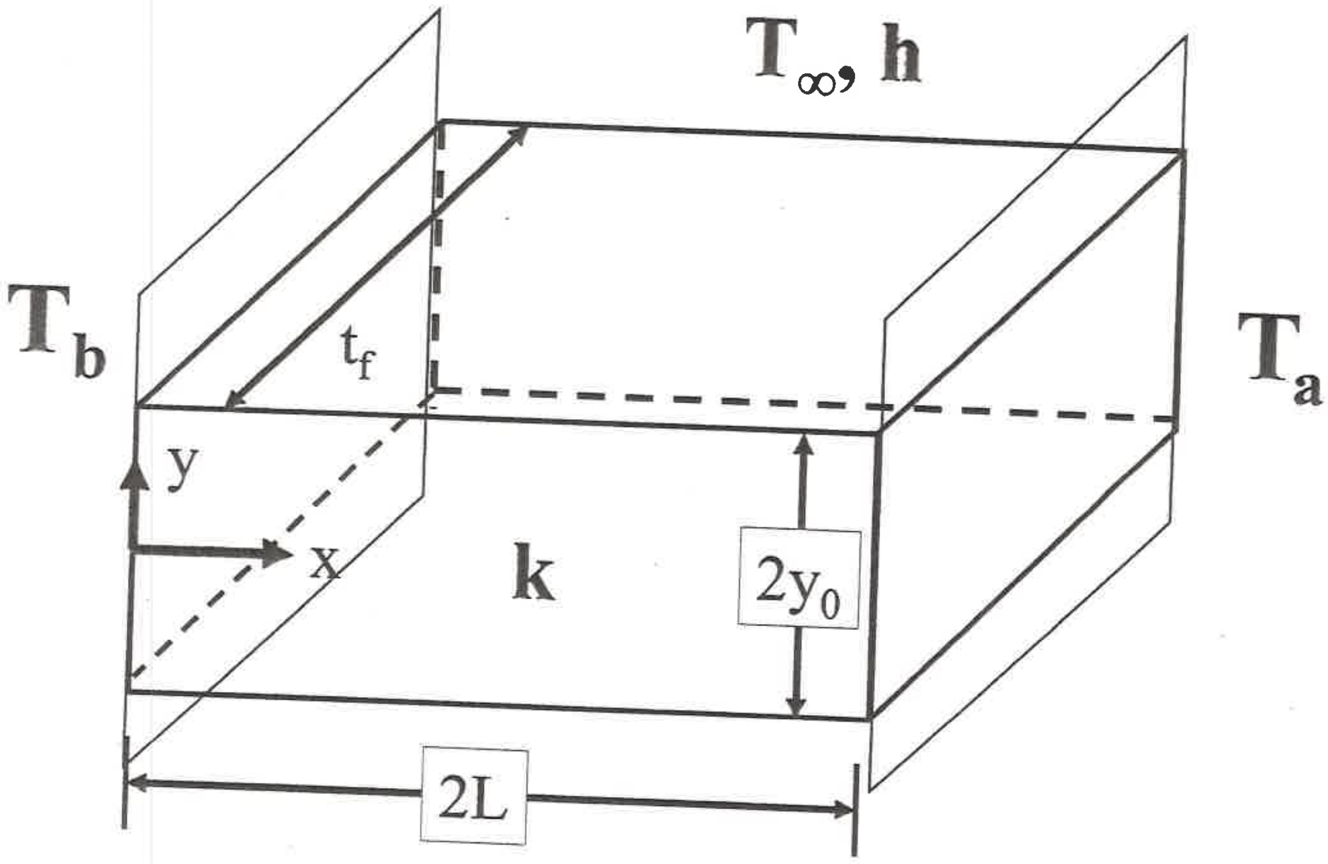
$2y_0$. The temperature and heat transfer coefficient of the ambient are T_∞ and h. Please find the temperature distribution in x-y plane. [*Hint*: please use the definition of dimensionless temperature $\theta = \frac{T - T_b}{T_\infty - T_b}$ to solve this problem]



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Supporting materials

(a) Continuity equation

$$\frac{\partial \rho}{\partial t}+\nabla \bullet \rho \vec{v}=0$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0 \text{ (Cartesian rectangular coordinates)}$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0 \text{ (Cylindrical coordinates)}$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi)=0 \text{ (Spherical coordinates)}$$

(b) Navier-Stokes equation

$$\rho\frac{D\vec{v}}{Dt}=-\nabla P+\mu\nabla^2\vec{v}+\rho\vec{g}$$

Cartesian Rectangular coordinates

$$\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_y$$

Cylindrical coordinates

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}\right)=-\frac{\partial P}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}\right)=-\frac{1}{r}\frac{\partial P}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$