國立交通大學 101 學年度碩士班考試入學試題

科目：統計學(4083)　　考試日期:101 年 2 月 17 日 第 3 節

系所班別：統計學研究所　　組別：統計所　　第 1 頁,共 1 頁

【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. Let X and Y be two random variables with joint pdf

$$f(x,y) = x + y,\ 0 < x < 1, 0 < y < 1.$$

(a) (5%) Please derive conditional expectation $E(Y|X)$ and verify that $E[E(Y|X)] = E(Y)$.

(b) (15%) Please verify that $Var(Y) = E[Var(Y|X)] + Var[E(Y|X)]$.

2 (20%). Let $X_1, ..., X_n$ be a random sample from $N(0, \theta^2)$, $\theta > 0$. Please derive the UMVUE of θ.

3. Let $X_1, ..., X_n$ be a random sample from a distribution with pdf

$$f(x, \theta) = e^{-(x-\theta)} I_{(\theta,\infty)}(x)$$

where $I_{(\theta,\infty)}(x) = 1$ if $\theta \le x < \infty$.

(a) (15%) Find a complete and sufficient statistic.

(b) (10%) Find the UMVUE of θ.

4. Let $Y_1, ..., Y_n$ be a random sample from a distribution with pdf $f(y) = \begin{cases} p_1 \text{ if } y = 1 \\ p_2 \text{ if } y = 2 \end{cases}$ with $p_1 + p_2 = 1$.

(a) (10%) Please derive the mgf of Y and the mean $E(Y)$ through this mgf.

(b) (10%) Let $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$. Please derive an approximate $100(1-\alpha)\%$ C.I. for p_1 (State the theorem applied in the derivation).

5 (15%). Please state and prove the Neyman-Pearson Theorem.