國立臺灣科技大學 112 學年度碩士班招生試題

系所組別：機械工程系碩士班丙組

科　　目：工程數學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. (10%) Find the general solution to the following ODE.

$$4y'''+4y''-y'-y=0$$

2. (10%) Please solve the following ODE.

$$\left(y^2-x^2\right)dx-(2xy)dy=0$$

3. (20%) A system ODE is shown below.

$$y_1'=y_1-4y_2+2y_3$$

$$y_2'=-2y_1-2y_2+2y_3$$

$$y_3'=4y_1+2y_2-y_3$$

$y_1(0)=1$, $y_2(0)=0$, and $y_3(0)=64$.

a) (18%) Find the eigenvalues and the corresponding eigenvectors.

b) (2%) Write the solution in vector form.

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4. (20%) Find the work done by a force $\vec{F}=x^2\,\vec{i}-2yz\,\vec{j}+z\,\vec{k}$ moves a particle along moves a particle along the straight line from $(1,1,1)$ to $(4,4,4)$.

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5. (20%) Solve the following 1D transient heat equation for the domain where $x > 0$ **using the Laplace transform**.

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t}$$

Boundary Condition 1: $T(x=0,t) = b + ct$

Boundary Condition 2: $T(x=\infty,t) = 0$

Initial Condition: $T(x,t=0) = 0$

where α is heat diffusivity (constant), and b and c are constants.

6. (20%) Answer the following questions.

a) (15%) Solve the partial differential equation

$\frac{\partial u}{\partial t} - c^2\frac{\partial^2 u}{\partial x^2} = 0,\ (0 < x < L,\ 0 \le t)$ subject to the boundary condition

$u(0,t) = u(L,t) = 0$ for $0 \le t$ and the initial conditions

$$u(x,0) = \begin{cases} x, & \text{for } 0 \le x \le \frac{L}{2} \\ L - x, & \text{for } \frac{L}{2} \le x \le L \end{cases}$$

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b) (5%) Determine the coefficients in the function of $u(x,t)$ based on the boundary and initial conditions.

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Laplace Table

$\bar{F}(s)$	$F(t)$
$\frac{1}{s^{\mu}}e^{-k/s}(\mu>0)$	$\left(\frac{t}{k}\right)^{(\mu-1)/2}J_{\mu-1}(2\sqrt{kt})$
$\frac{1}{s^{\mu}}e^{-k/s}(\mu>0)$	$\left(\frac{t}{k}\right)^{(\mu-1)/2}I_{\mu-1}(2\sqrt{kt})$
$e^{-k\sqrt{s}}(k>0)$	$\frac{k}{2\sqrt{\pi t^3}}\exp\left(-\frac{k^2}{4t}\right)$
$\frac{1}{s}e^{-k\sqrt{s}}(k\geqslant 0)$	$\operatorname{erfc}\frac{k}{2\sqrt{t}}$
$\frac{1}{\sqrt{s}}e^{-k\sqrt{s}}(k\geqslant 0)$	$\frac{1}{\sqrt{\pi t}}\exp\left(-\frac{k^2}{4t}\right)$
$\frac{e^{-k\sqrt{s}}}{\sqrt{s}(a+\sqrt{s})}(k\geqslant 0)$	$e^{ak}a^{a^2t}\operatorname{erfc}\left(a\sqrt{t}+\frac{k}{2\sqrt{t}}\right)$
$\frac{e^{-k\sqrt{s(s+a)}}}{\sqrt{s(s+a)}}(k\geqslant 0)$	$e^{-at/2}I_0(\frac{1}{2}a\sqrt{t^2-k^2})u(t-k)$
$\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}(k\geqslant 0)$	$J_0(a\sqrt{t^2-k^2})u(t-k)$
$\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2-a^2}}(k\geqslant 0)$	$I_0(a\sqrt{t^2-k^2})u(t-k)$
$\frac{ae^{-k\sqrt{s}}}{s(a+\sqrt{s})}(k\geqslant 0)$	$-e^{ak}e^{a^2t}\operatorname{erfc}\left(a\sqrt{t}+\frac{k}{2\sqrt{t}}\right)+\operatorname{erfc}\frac{k}{2\sqrt{t}}$
$\frac{1}{s^2}e^{-k\sqrt{s}}$	$\left(t+\frac{k^2}{2}\right)\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)-k\left(\frac{t}{\pi}\right)^{1/2}\exp\left(-\frac{k^2}{4t}\right)$

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