

國立臺灣科技大學 112 學年度碩士班招生試題

系所組別：機械工程系碩士班甲組

科目：工程數學

(總分為 100 分；所有試題務必於答案卷內頁依序作答，否則不予計分)

1. (20%) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(a) (10%) Find the inverse matrix of A

(b) (10%) Find the LU decomposition of A, which means $A=LU$

2. (15%) $t - 2f(t) = \int_0^t (e^\tau - e^{-\tau})f(t - \tau) d\tau$, solve $f(t)=?$

3. (15%) $y'' - 3y' = 8e^{3x} + 4$, solve $y=?$

4. (10%) Compute the directional derivative of the field $u(x, y, z) = 2x^2 + 3y^2 + z^2$ in the direction of the vector $\vec{i} - 2\vec{k}$ at the point P: (2, 1, 3).

5. (20%) $\vec{F} = 7x\vec{i} - z\vec{k}$

(a) (10%) Compute $\iint_S (\vec{F} \cdot \vec{n}) dA$ over the sphere $S : x^2 + y^2 + z^2 = 4$ directly.(b) (10%) Check the result of $\iint_S (\vec{F} \cdot \vec{n}) dA$ over the sphere $S : x^2 + y^2 + z^2 = 4$ by divergence theorem.

6. (20%) Starting from separation of variables, solve the following partial differential equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \quad (c \text{ is a constant.})$$

Boundary conditions : $u(0, t) = u(L, t) = 0$ for all $t \geq 0$ Initial conditions : $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x, 0) = 0$ ($0 \leq x \leq L$).

where

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$$

