國立中央大學112學年度碩士班考試入學試題

所別： 地球科學學系地球物理碩士班

科目： 微積分

作答時須列出完整計算過程

1. (a) (5%) $\lim_{x\to a} \frac{a-x}{ln\frac{x}{a}} = ?$

   (b) (5%) $\lim_{x\to\infty} \frac{3x+4}{\sqrt{2x^2-5}} = ?$

2. (a) (5%) $\frac{d}{dx}(tanh(x)) = ?$ [*tanh(x)* is a hyperbolic function.]

   (b) (5%) $\frac{d}{dx}(sin^{-1}x) = ?$

3. (a) (5%) $\int e^{ax} sinbx dx = ?$

   (b) (5%) $\int_{-\infty}^{\infty} e^{-x^2} dx = ?$

4. (10%) Find a general solution.

$$y'' + 3y' + 2y = 12x^2.$$

5. (10%) Use the method of separating variables to solve the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, for the vibrations of an elastic string of length L.
   The boundary conditions are $u(0,t) = 0,\ u(L,t) = 0$ for all t. The initial conditions are $u(x,0) = f(x),\ u_t(x,t)|_{t=0} = 0$.

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6. (10%) Find the **odd** periodic expansions of the function (half-range expansion)

$$f(x)=\begin{cases}\dfrac{2k}{L}x & \text{if} \quad 0<x<\dfrac{L}{2}\\ \dfrac{2k}{L}(L-x) & \text{if} \quad \dfrac{L}{2}<x<L.\end{cases}$$

7. (10%) Find the inverse of $A=\begin{bmatrix}-1 & 1 & 2\\ 3 & -1 & 1\\ -1 & 3 & 4\end{bmatrix}$

8. (10%) Experiments show that at each instant a radioactive substance decays at a rate proportional to the amount present. Show that $\lambda T_{1/2}=\ln 2$, where λ is decay constant and $T_{1/2}$ is "half-life", period of time during which the radioactive substance decays to half.

9. (a) (4%) Explain thermal conductivity and specific heat.

   (b) (2%) What is divergence theorem?

   (c) (4%) Model heat flow from a body in space to derive heat equation.

10. (10%) 證明半徑為 a 之圓，其周長為 $2\pi a$

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