國立成功大學

112學年度碩士班招生考試試題

編　號：55

系　所：太空與電漿科學研究所

科　目：應用數學

日　期：0206

節　次：第 2 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Consider a force $\mathbf{F} = -\dfrac{y}{x^2+y^2}\hat{\imath} + \dfrac{x}{x^2+y^2}\hat{\jmath}$.

   (a) (5%) Please calculate $\nabla \times \mathbf{F}$.

   (b) (5%) Is $\mathbf{F}$ a conservative force? Why?

   (c) (10%) Calculate $\oint_C \mathbf{F} \cdot d\mathbf{l}$, where C is the unit circle $x^2 + y^2 = 1$.

2. Consider an integral equation: $y(t) = t + \int_0^t \sin(t-\tau)\,y(\tau)d\tau$.

   (a) (10%) Please use Laplace transform to find $y(t)$.

   (b) (10%) Please convert the integral equation into a differential equation by taking $\dfrac{d^2}{dt^2}y(t) =$

   $\dfrac{d^2}{dt^2}\Big[t + \int_0^t \sin(t-\tau)\,y(\tau)d\tau\Big]$, and then find $y(t)$.

3. Consider a coupled differential equation: $\begin{cases} \dot{y}_1(t) = 5y_1 + 8y_2 + 1 \\ \dot{y}_2(t) = -6y_1 - 9y_2 + t \end{cases}$, with initial conditions $\begin{cases} y_1(0) = 4 \\ y_2(0) = -3 \end{cases}$.

   (a) (10%) Please decouple it as two differential equations.

   (b) (10%) Please find $y_1(t)$ and $y_2(t)$.

4. (10%) Please find $\int_{-\infty}^{\infty} \dfrac{\sin x}{x}dx$. [Hint: $\oint \dfrac{e^{iz}}{z}dz$]

5. A and B are invertible matrix,

   (a) (5%) please show that $(AB)^{-1} = B^{-1}A^{-1}$.

   Consider $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$,

   (b) (5%) Find A^{-1}.

   (c) (5%) Find B^{-1}.

   (d) (5%) Find AB

   (e) (5%) Find $(AB)^{-1}$

   (f) (5%) Please verify $(AB)^{-1} = B^{-1}A^{-1}$ by the given two matrix.