

國立高雄第一科技大學 100 學年度 碩士班 招生考試 試題紙

系所別：風險管理與保險系

組別：精算資訊組

考科代碼：2424

考科：微積分

注意事項：

- 1、本科目得使用本校提供之電子計算器。
- 2、請於答案卷上規定之範圍作答，違者該題不予計分。

請寫出詳細計算過程並依序作答。

1. Analysis the graph of $f(x) = \frac{x^2 - 2x + 4}{x - 2}$.

- (1) Define the domain. (4 points)
- (2) Find the intercepts. (4 points)
- (3) Find the relative extrema. (4 points)
- (4) Find the asymptotes. (4 points)
- (5) Find the inflection points. (4 points)

2. Find $\int_1^e x^5 \ln x dx$. (10 points)

3. Compute the exact value then use the Simpson's Rule to approximate the value of the definite integral $\int_0^2 (x^4 + 1) dx$ for $n = 4$. Round your answers to four decimal places. (10 points)

The Simpson's Rule: If f is continuous on $[a, b]$, then

$$\int_a^b f(x) dx \approx \left(\frac{b-a}{3n} \right) [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 4f(x_{n-1}) + f(x_n)]$$

4. Sketch and find the area of the region bounded by the graph of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$. (10 points)

5. Decide whether the following series converges or diverges?

(1) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$. (5 points)

(2) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$. (5 points)

(3) $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n$. (5 points)

(4) $\sum_{n=0}^{\infty} \frac{2^n}{n!}$. (5 points)

6. If $\int_a^c f(x)dx = 1$, $\int_a^b f(x)dx = 2$, $\int_b^d g(x)dx = 4$, and $\int_c^d g(x)dx = -5$, then calculate

$\int_b^c [3f(x) - 4g(x)]dx$. (10 points)

7. Find $\int_0^1 \int_0^1 x^3 ye^{x^2y^2} dx dy$. (10 points)

8. Let f and g be functions differentiable on the interval $(0, 5)$ with the following values for the functions and their derivatives:

x	1	2	3	4
$f(x)$	1	3	4	2
$f'(x)$	1	2	3	4
$g(x)$	2	4	1	3
$g'(x)$	3	1	4	2

Determine the derivative of the composite function $g(f(x))$ at $x = 2$. (10 points)