國立高雄第一科技大學 100 學年度 碩士班 招生考試 試題紙

系 所 別：風險管理與保險系　　組　別：精算資訊組

考科代碼：1423　　考　科：統計學

注意事項：

1、**本科目得使用本校提供之電子計算器。**

2、請於答案卷上規定之範圍作答，違者該題不予計分。

請寫出詳細計算過程並依序作答。

1. Machine consists of two components. Let X be the random variable representing the lifetime of the first component, and let Y represent the lifetime of the second component. X and Y have joint density function

$$f(x,y)=\begin{cases}k, & \text{for } x>0, y>0, x+2y<6\\ 0, & \text{otherwise,}\end{cases}$$

where k is a constant. The machine operates until both components fail. Let T be the time that the machine is out of operation.

(1) Evaluate the constant k. (*5 points*)

(2) Determine the probability $P(T\le 1)$. (*5 points*)

(3) Determine the probability $P\left(T\le \frac{5}{2}\right)$. (*5 points*)

(4) Determine the probability $P(T\le 4)$. (*5 points*)

(5) Find the density of T. (*10 points*)

(6) Calculate the expectation of T. (*10 points*)

2. If the cumulative distribution function of a random variable X is given by

$$F(x)=\begin{cases}0, & x<0\\ 1-\frac{1}{3}e^{-2x}, & x\ge 0\end{cases},$$

then what is its moment generating function? (*10 points*)

3. Let $P(A \cap B) = 0.3$, $P(A) = 0.6$, $P(B) = 0.5$. Find $P(A' \cap B')$. (*5 points*)

4. Let the random variable have the p.d.f.

$$f(x) = \sqrt{\frac{2}{\pi}} e^{-\frac{x^2}{2}}, \text{ when } 0 < x < \infty, \text{ and zero elsewhere.}$$

(1) Find the mean. (*10 points*)

(2) Find the variance. (*10 points*)

5. A large number of insects are expected to be attracted to a certain variety of rose plant. A commercial insecticide is advertised as being 99% effective. Suppose 2,000 insects infest a rose garden where the insecticide has been applied, and let X = number of surviving insects.

(1) What probability distribution might provide a reasonable model for this experiment? (*5 points*)

(2) Write down, but do not evaluate, an expression for the probability that fewer than 29 insects survive, using the model in part (1). (*10 points*)

(3) Using Normal distribution to evaluate an approximation to the probability. (*10 points*)

$P(0 < Z < 1.11) = 0.3665$, $P(0 < Z < 1.51) = 0.4345$, $P(0 < Z < 1.91) = 0.4719$,
$P(0 < Z < 2.31) = 0.4896$, $P(0 < Z < 2.71) = 0.4966$, $P(0 < Z < 3.01) = 0.4987$