Show all your work and carefully justify all your answers. Answers without explanation will not receive any score.

1. (10 分) If $f(x) = \int_{\arctan x}^{x} x^2 \sin(t^2)\, dt$, find $f'(x)$.

2. (12 分) Find the Maclaurin series for $f(x) = x^2 \arctan(x^3)$ and its radius of convergence.

3. (12 分) Find the extreme values of $f(x,y) = e^{-x^2-y^2}(x^2 + 2y^2)$ on the disk $x^2 + y^2 \le 4$.

4. Consider the function

$$f(x,y) = \begin{cases} \dfrac{x^3 y - x y^3}{x^2 + y^2} & \text{if } (x,y) \ne (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) (10 分) Determine whether the limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists. Prove or disprove your answer.

   (b) (12 分) Show that $f_{xy}(0,0) \ne f_{yx}(0,0)$.

5. (10 分) Evaluate the integral

$$\int_0^1 \int_0^1 e^{\max\{x^2, y^2\}}\, dy\, dx$$

   where $\max\{x^2, y^2\}$ means the larger of the numbers x^2 and y^2.

6. (10 分) Evaluate the line integral

$$\int_C y^3\, dx - x^3\, dy$$

   where $C = \{(x,y) \mid x^2 + y^2 = 4\}$ is traversed counterclockwise.

7. (12 分) If $0 < a < b$, find

$$\lim_{x\to 0} \left\{ \int_0^1 [bt + a(1-t)]^x\, dt \right\}^{1/x}.$$

8. (12 分) Find the area of the region that lies inside the polar curve $r = 3\cos\theta$ and outside the polar curve $r = 1 + \cos\theta$.