

國立中正大學

112 學年度碩士班招生考試

試題

[第 1 節]

科目名稱	工程數學
系所組別	機械工程學系-乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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系所組別：機械工程學系-乙組

1. (30%) Use Laplace transform to solve the following second order differential equation

$$\frac{d^2 f}{dx^2} + af = u, \quad x \geq 0$$

with initial values $f(0) = f_0, \frac{df(0)}{dx} = f_1$

- (a) (20%) Assume $a = -\alpha^2 < 0$ and $u = 0, f_0 = 1, f_1 = 2$. Determine $f(x), x \geq 0$.
 (b) (10%) Assume $a = \omega^2 > 0$ and $u = \sin(\omega x), f_0 = 0, f_1 = 0$. Determine $f(x), x \geq 0$.

2. (20%) Consider the boundary-value problem of the following fourth order differential equation

$$\frac{d^4 f}{dx^4} - \beta^4 f = 0, \quad 0 \leq x \leq 1$$

where β is a parameter to be determined.

- (a) (10%) Show that the general form of the solution is

$$f(x) = A \cos(\beta x) + B \sin(\beta x) + C \cosh(\beta x) + D \sinh(\beta x)$$

- (b) (10%) Determine the possible β 's and corresponding non-zero solutions when the boundary values are $f(0) = 0, f(1) = 0, \frac{d^2 f(0)}{dx^2} = 0, \frac{d^2 f(1)}{dx^2} = 0$.

3. (35%) Consider a rigid body with 2-DOF (degree-of-freedom) rotational motion whose mass moment of inertia matrix is given by

$$I = \begin{bmatrix} I_x & -I_{xy} \\ -I_{xy} & I_y \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix}$$

- (a) (25%) Please find the principal mass moments of inertia (eigenvalues of I) and principal axes (eigenvectors of I).
 (b) (10%) Now there is an applied torque given by $\tau = [3 \quad -5]^T$. Suppose that the unit vectors of the principal axes are e_1 and e_2 . Please represent the applied torque τ using the coordinate formed by the principal axes. That is, for $\tau = ae_1 + be_2$, please find a and b .

4. (15%) Please answer the following questions.

- (a) (10%) Please explain Fourier series and Fourier transform. You can use examples to help you on answering this question.
 (b) (5%) Please describe a practical example that utilizes either Fourier series or Fourier transform.