國立中正大學
112 學年度碩士班招生考試
試 題

［第 2 節］

科目名稱	線性代數
系所組別	數學系應用數學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

Problem 1 (12 points) Find an orthogonal basis for the row space of the matrix

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 4 & -10 & 2 \\ 4 & -1 & 2 \\ -2 & 14 & -1 \end{bmatrix}$$

Problem 2 (10 points) Let $T_1 : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by:

$$T_1\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ 3x_2 - 4x_3 \end{bmatrix}$$

and let $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ be the geometric linear transformation which refects a point $x = (x_1, x_2) \in \mathbb{R}^2$ across the line $x_1 = x_2$. Find the standard matrix A of the composition $T = T_2^{-1} \circ T_1$.

Problem 3 (14 points) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 4 & 3 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$.

(a) (5 points) Find a basis for the column space of A.

(b) (5 points) Find a basis for the null space of A.

(c) (4 points) Find complete solution to $Ax = \begin{bmatrix} 8 \\ 4 \\ 2 \end{bmatrix}$.

Problem 4 (12 points) Find the minimal polynomial $p(x)$ of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & -1 & 3 \end{bmatrix}$$

and use $p(x)$ to find A^{-1}.

Problem 5 (12 points) Let $B = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$, $O_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and A is the block matrix $\begin{bmatrix} B & O_2 \\ O_2 & C \end{bmatrix}$.

(a) (6 points) Find orthogonal matrix Q such that $Q^T BQ$ is a diagonal matrix.

(b) (6 points) Compute A^{50}.

Problem 6 (40 points) Determine the following statements are true or false. Please give a proof if true or counterexample if false.

(a) (8 points) If A is similar to B and B is orthogonal, then A must be orthogonal.

(b) (8 points) All 3×3 skew symmetric matrices are singular (Note: A is skew symmetric if $A^T = -A$).

(c) (8 points) The set of 2×2 diagonalizable real matrices is a subspace of the set of all 2×2 real matrices, with scalar multiplication and matrix addition defind entrywise.

(d) (8 points) If A is an $m \times n$ matrix, then the null space of $A^T A$ is equal to the null space of A.

(e) (8 points) If $L : \mathbb{R}^6 \to M_{23}$ is a linear transformation which is onto, then L is invertible.