國立中正大學
112學年度碩士班招生考試
試題
[第1節]

科目名稱	微積分
系所組別	數學系應用數學

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始40分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1. Find the following limits, derivatives or integrals: (Each problem: 6%)

   (a) If f is a continuous function on $[0, \infty)$ and satisfies
   $$\int_0^{x^2} f(t)dt = x\ln(x+1),$$
   then find $f(1)$.

   (b) Evaluate
   $$\lim_{x\to\infty} x^4(\cos\frac{1}{x} - 1 + \frac{1}{2x^2}).$$

   (c) Evaluate
   $$\lim_{n\to\infty} \frac{1^{111} + 2^{111} + \ldots + n^{111}}{n^{112}}.$$

   (d) Evaluate
   $$\int_1^3 \sqrt{4x - x^2}dx.$$

   (e) Evaluate
   $$\int_0^3 \int_x^3 e^{-y^2} dydx.$$

2. Find the area of the region bounded by the graph of $y = \frac{e^{2x}}{(e^{2x}+1)(e^x-1)}$ and three lines: $y = 0, x = \ln 2$ and $x = \ln 3$ in $\mathbf{R}^2$. (10%)

3. Prove or disprove the following statement: the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ converges conditionally. (10%)

4. Set
   $$f(x,y) = \begin{cases} \frac{xy^2}{x^3+y^3} & \text{if } (x,y) \neq (0,0). \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) Does $f_y(0,0)$ exist? (5%)

   (b) Is f differentiable on $\mathbf{R}^2$? (5%)

5. Suppose $z = f(x,y)$, where $x = x(r,\theta) = r\cos\theta$ and $y = y(r,\theta) = r\sin\theta$.

   (a) Find $\frac{\partial z}{\partial r}$. (5%)

   (b) Show that $(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2$. (5%)

6. Find the maximum value of the directional derivative of

$$f(x,y,z) = x^2 e^{yz}$$

at $(2,-2,0)$. (10%)

7. Find

$$\iint_R \frac{3x^2y^2}{1+x^3y^3} dA,$$

where R is the region bounded by $x=1, x=4, xy=1$ and $xy=4$ in $\mathbf{R^2}$.(10%)

8. Set

$$\mathbf{F} = \left(\frac{-4y}{x^2+y^2}, \frac{4x}{x^2+y^2} \right).$$

Find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the positive oriented curve for $\frac{(x-10)^2}{9} + \frac{(y-10)^2}{16} = 1$. (10%)