

類組：電機類 科目：離散數學(300C)

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6. [10 points] Dr. DM represents a relation \star on the set of functions from positive integer \mathbb{N}^+ to \mathbb{R} , such that $f \star g$ if and only if $f = O(g)$. Is \star an equivalence relation? A partial order? A total order? Briefly explain your answer.

7. [10 points] Suppose a and b are relatively prime integers greater than 1. Please help Dr. DM prove that Euler's function is multiplicative. In other words, $\phi(ab) = \phi(a)\phi(b)$. Note that Euler's Totient function $\phi(n)$ for an input n is the count of numbers in $1, 2, 3, \dots, n-1$ that are relatively prime to n , i.e., the numbers whose GCD (Greatest Common Divisor) with n is 1.

8. [12 points (4 each)] After thousands of years, NYCU nation has developed many family trees. Let P denote the set of all people who ever lived. $Parent(x, y)$ is true if and only if x is the parent of y . $Ancestor(x, y)$ is true if and only if x is the ancestor of y . $Fight(x, y)$ is true if and only if x has a fight with y . Rewrite in the language of mathematical logic. The operations are limited to $\forall, \exists, \neq, \in, \rightarrow, \wedge, \vee, \leftrightarrow$, and \neg .

(a) All people have two parents.

(b) An ancestor of a person is one of the person's parents or the ancestor of (at least) one of the person's parents. Rewrite this definition using the language of mathematical logic. Specifically, you need to provide a necessary and sufficient condition for the predicate $Ancestor(x, y)$ to be true. (Note that you can inductively use the $Ancestor(\cdot, \cdot)$ predicate in the condition itself.)

(c) Someone has a fight with his/her ancestor but does not have a fight with his/her offspring.

注意:背面有試題