國立交通大學 101 學年度碩士班考試入學試題

科目：離散數學(4041) 考試日期:101 年 2 月 17 日 第 1 節

系所班別：應用數學系 組別：應數系乙組 第 1 頁,共 1 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

每題都會依據作答情形給部份分數, 請勿因想法不夠完美就放棄該題, 另一方面, 要有完整的解釋, 並符合邏輯, 才能得到全部分數.

1. (10) Two overlapping circles create 4 regions. Three overlapping circles create 8 regions. How many regions the most that four mutually overlapping circles in the plane can create?

2. (10) There are 20 identical sticks lined up in a row occupying 20 distinct places as follows:

   $$||||||||||||||||||||.$$

   Six of them are to be chosen. How many choices are there if no two of the chosen sticks can be consecutive?

3. (10) Let a_n be the number of sequences of length n over $\{R, L, U\}$ without consecutive subsequences RL and LR. Note that each such sequence must ends with U, LL, RR, UL, UR. Define $a_0 = 1$ and clearly $a_1 = 3$. Write a recursion relation for a_n for $n \geq 2$.

4. Let $\phi(n)$ denote the number of positive integers not exceeding n that are relatively prime to n. For example $\phi(12) = |\{1, 5, 7, 11\}| = 4$.

   (a) (5) Show that $12 = \phi(1) + \phi(2) + \phi(3) + \phi(4) + \phi(6) + \phi(12)$.

   (b) (10) Show that $n = \sum_{k|n} \phi(k)$ for $n \in \mathbb{N}$.

   (c) (10) Find the 6×6 matrix μ such that

   $$(\phi(1), \phi(2), \phi(3), \phi(4), \phi(5), \phi(6)) = (1, 2, 3, 4, 5, 6)\mu.$$

5. Let $A_1, A_2, \ldots, A_m$ be subsets of S, where S is a finite set.

   (a) (10) Suppose $A_i \cap A_j \cap A_k \cap A_\ell = \emptyset$ for any $1 \leq i < j < k < \ell \leq m$. Show that

   $$\sum_{1 \leq i \leq m} |A_i| - \sum_{1 \leq i < j \leq m} |A_i \cap A_j| + \sum_{1 \leq i < j < k \leq m} |A_i \cap A_j \cap A_k| = |A_1 \cup A_2 \cup \cdots \cup A_m|.$$

   (b) (15) Suppose

   $$\sum_{1 \leq i \leq m} |A_i| - \sum_{1 \leq i < j \leq m} |A_i \cap A_j| + \sum_{1 \leq i < j < k \leq m} |A_i \cap A_j \cap A_k| = |A_1 \cup A_2 \cup \cdots \cup A_m|.$$

   Show that $A_i \cap A_j \cap A_k \cap A_\ell = \emptyset$ for any $1 \leq i < j < k < \ell \leq m$.

6. (20) Let G be a connected graph of order n. Show that there exists a vertex in G whose deletion will not disconnect G.