

國立成功大學

112學年度碩士班招生考試試題

編 號：103

系 所：土木工程學系

科 目：工程統計

日 期：0206

節 次：第 3 節

備 註：可使用計算機

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let  $X$  be a discrete random variable with a cumulative distribution function (cdf) shown below. (5 points each)

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.30 & 1 \leq x < 3 \\ 0.40 & 3 \leq x < 4 \\ 0.45 & 4 \leq x < 6 \\ 0.60 & 6 \leq x < 12 \\ 1 & 12 \leq x \end{cases}$$

- (1) What is the probability mass function (pmf) of  $X$ ?
- (2) What is  $P(3 \leq X \leq 6)$ ?
- (3) What is  $P(4 \leq X)$ ?
- (4) What is the variance of  $X$ ?

2. The weekly demand for water (in 100,000s of tons) from a particular city is a random variable of  $X$  with the probability density function (pdf) as follows: (5 points each)

$$f(x) = \begin{cases} 2 \left(1 - \frac{1}{x^2}\right) & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (1) What is the cdf of  $X$ ?
- (2) What is the expected value of  $X$ ?
- (3) What is the variance of  $X$ ?
- (4) Obtain an expression for the (100p)th percentile. What is the median value of  $X$ ?

3. The following summary provides information for the fracture strengths (MPa) of  $n = 169$  metal bars manufactured in a particular process:  $\bar{x} = 89.1$ ,  $s = 3.73$ . (5 points each)

- (1) What is the confidence interval (CI) for true average fracture strength with a confidence level of 95%?
- (2) Does it appear that true average fracture strength is precisely estimated using the above interval? Why?
- (3) What are the ways to reduce the margin of error for CI?
- (4) Suppose the investigators had believed a priori that the population standard deviation was about 4 MPa. Based on this supposition, how large a sample would have been required to estimate  $m$  to within 0.5 MPa with 95% confidence?

Values Provided for Your Calculations

$z$	1.64	1.96	2.58	$t_{0.025, 5}$	$t_{0.025, 4}$	$t_{0.05, 5}$	$F_{0.05, 2, 3}$	$F_{0.05, 3, 2}$	$F_{0.025, 4, 2}$	$F_{0.025, 4, 3}$	$F_{0.025, 3, 2}$	$F_{0.025, 3, 4}$
$\Phi(z)$	0.95	0.975	0.995	2.571	2.776	2.015	19.00	19.16	39.25	15.10	39.12	9.98

4. The measurements on viscosity (cP) for specimens of a certain grade of highway materials are as follows: 2781, 2900, 3013, 2856, and 2888. Suppose that for a particular application it is required that true average viscosity be 3000. Note:  $\bar{x} = 2887.6$ ,  $s = 84$ . (5 points each)

- (1) State the relevant hypotheses.
- (2) Calculate the test statistic.
- (3) State the rejection region using  $\alpha=0.05$ .
- (4) Make your conclusion. Does this requirement appear to have been satisfied?

5. The physical properties of a certain type of soils are analyzed using a quadratic regression of  $y =$  bulk density (i.e., bulkdens) on  $x =$  moisture content (i.e., moiscont). The following output for the quadratic model is obtained from software called Minitab. Note: The quadratic predictor is named as contsqd. (5 points each)

The regression equation is

$$\text{bulkdens} = 403 + 16.2 \text{ moiscont} - 0.706 \text{ contsqd}$$

Predictor	Coef	StDev	T	P
Constant	403.24	36.45	11.06	0.002
moiscont	16.164	5.451	2.97	0.059
contsqd	-0.7063	0.1852	-3.81	0.032

S = 10.15      R-Sq = 93.8%      R-Sq(adj) = 89.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	4637.7	2318.9	22.51	0.016
Residual Error	3	309.1	103.0		
Total	5	4946.8			

Obs	moiscont	bulkdens	Fit	StDev	Fit Residual	St Resid
1	7.0	479.00	481.78	9.35	-2.78	-0.70
2	10.3	503.00	494.79	5.78	8.21	0.98
3	13.7	487.00	492.12	6.49	-5.12	-0.66
4	16.6	470.00	476.93	6.10	-6.93	-0.85
5	19.8	458.00	446.39	5.69	11.61	1.38
6	22.0	412.00	416.99	8.75	-4.99	-0.97

Fit	StDev	Fit	95.0% CI	95.0% PI
491.10	6.52	(470.36, 511.83)	(452.71, 529.48)	

- (1) What proportion of observed variation in density can be explained by the quadratic model?
- (2) State the relevant hypotheses for the quadratic model.
- (3) Carry out a test of hypotheses at significance level 0.05 to decide if the quadratic model provides useful information according to the above output.
- (4) Does the quadratic predictor appear to provide useful information? State and test the appropriate hypotheses at significance level 0.05.